

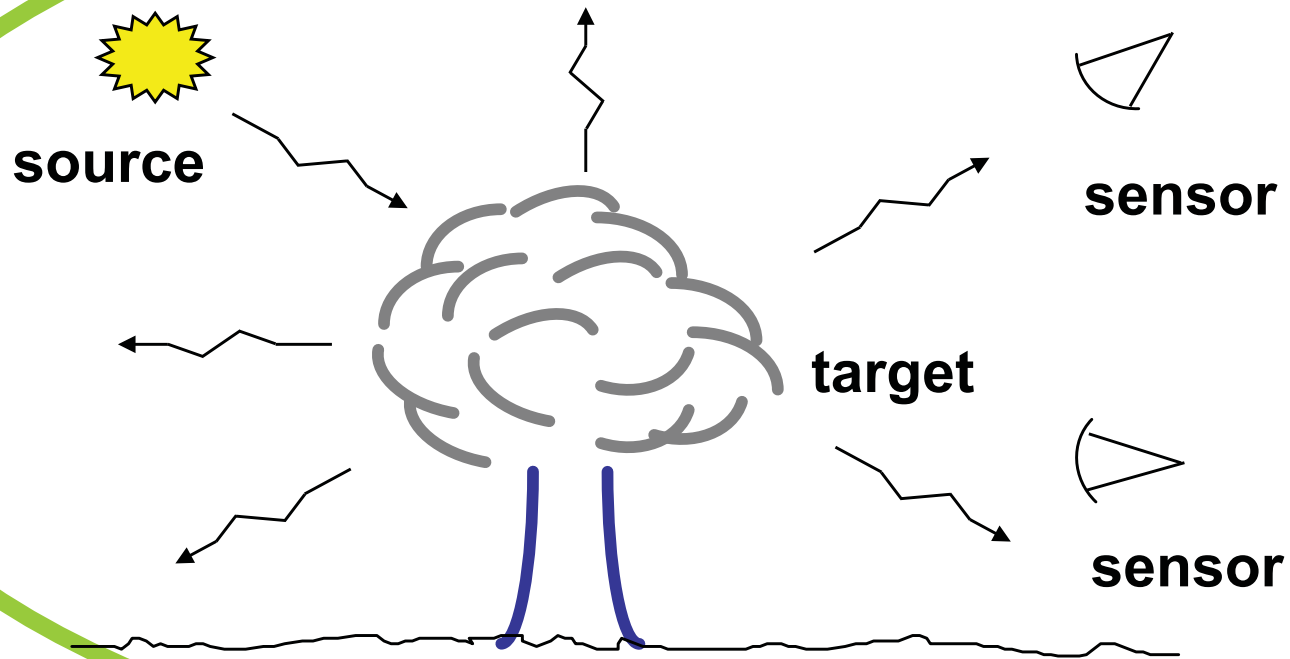


# **Introduction to radiative transfer theory and models (Optical Domain)**

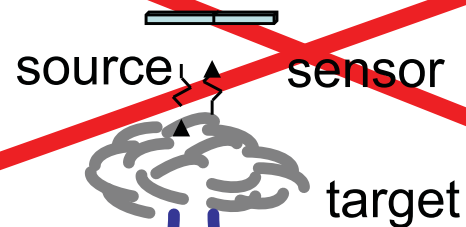
**Dr. Eric Vermote  
NASA GSFC Code 619  
[Eric.f.vermote@nasa.gov](mailto:Eric.f.vermote@nasa.gov)**



## Passive Optical Remote Sensing



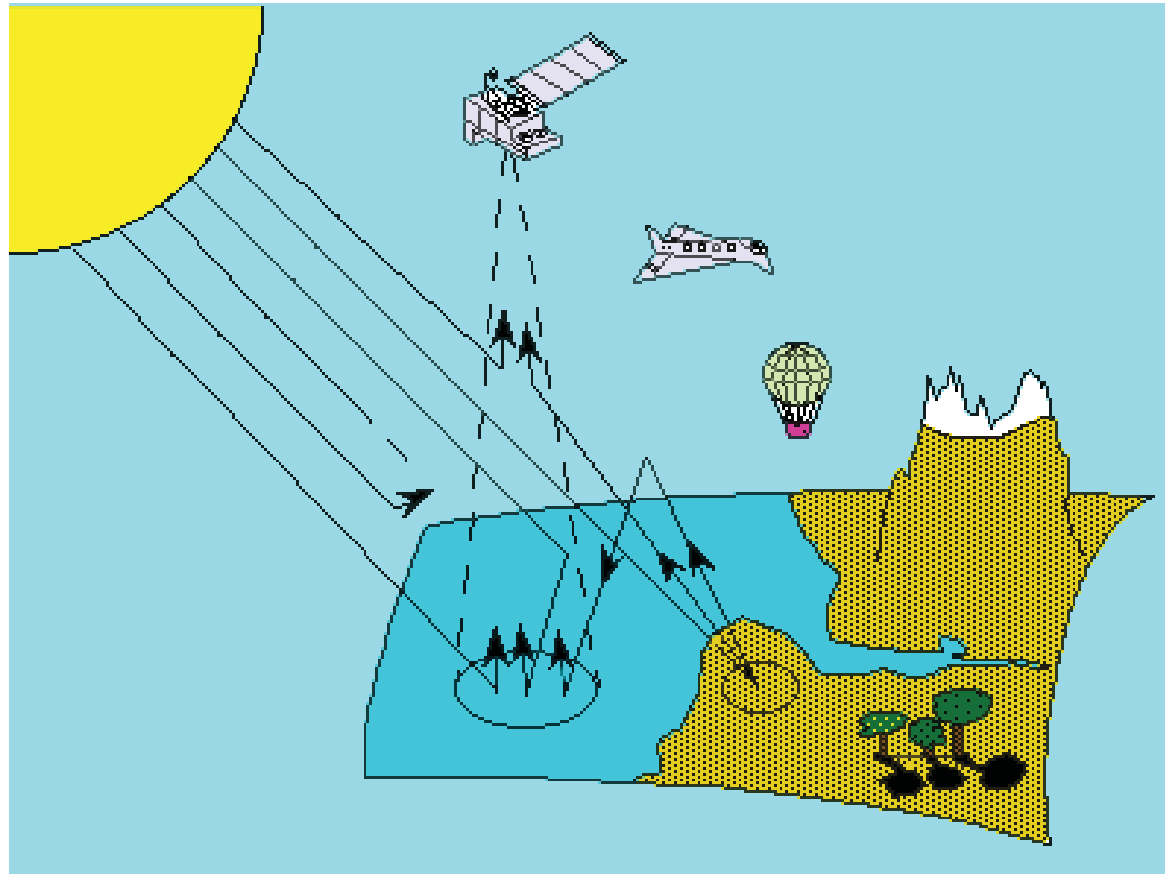
RADAR: radiation generated next to sensor

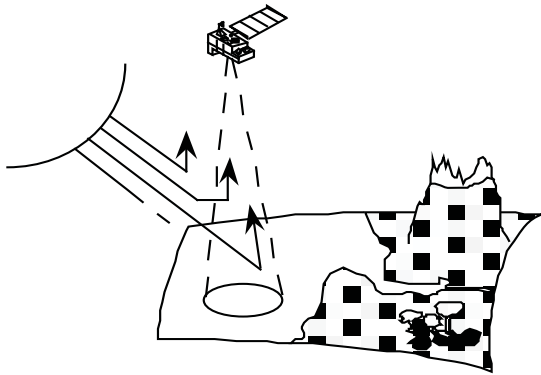


Atmospheric Correction of Earth-  
Observation Data for  
Environmental Monitoring: Theory

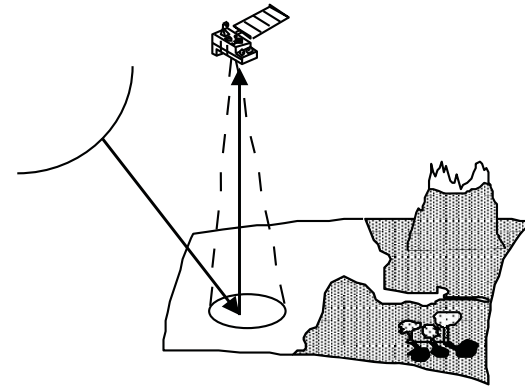


# Solar Energy Paths

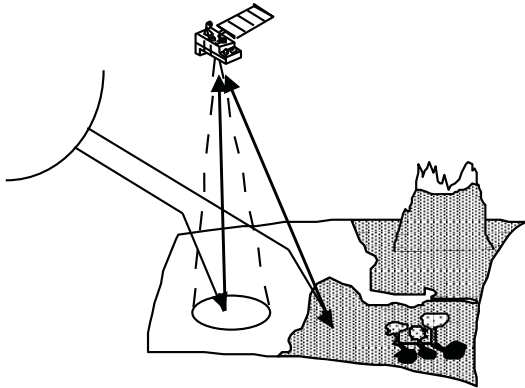




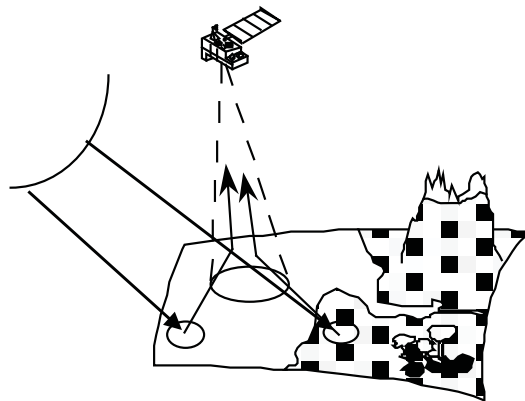
atmospheric contribution



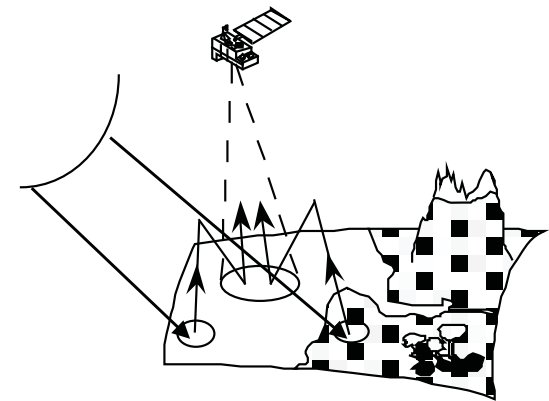
direct + direct



diffuse + direct



direct + diffuse

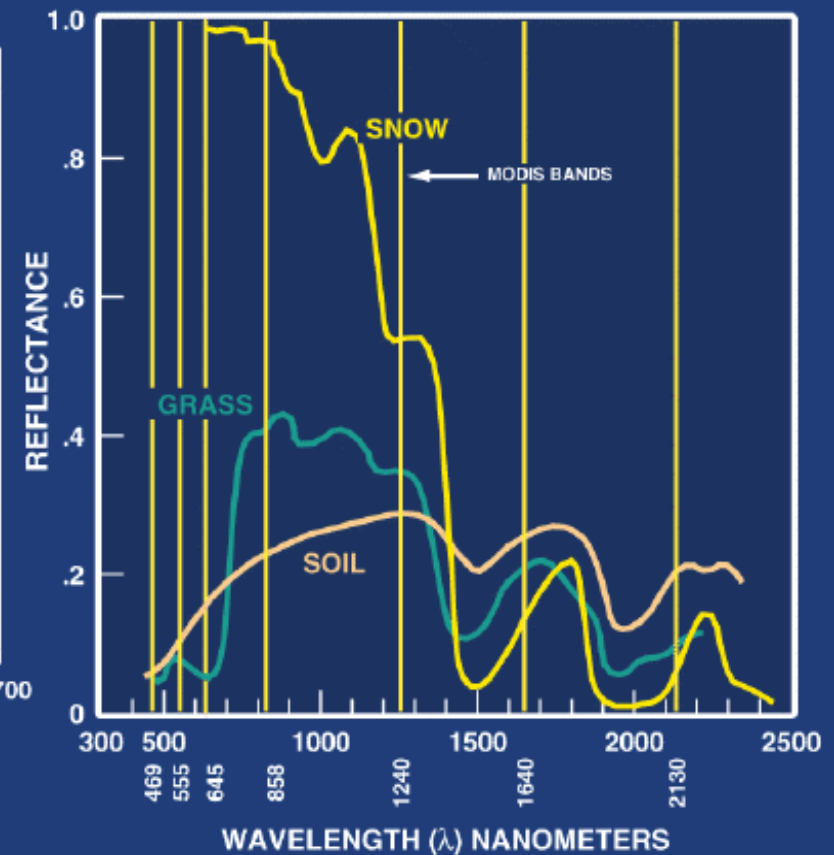
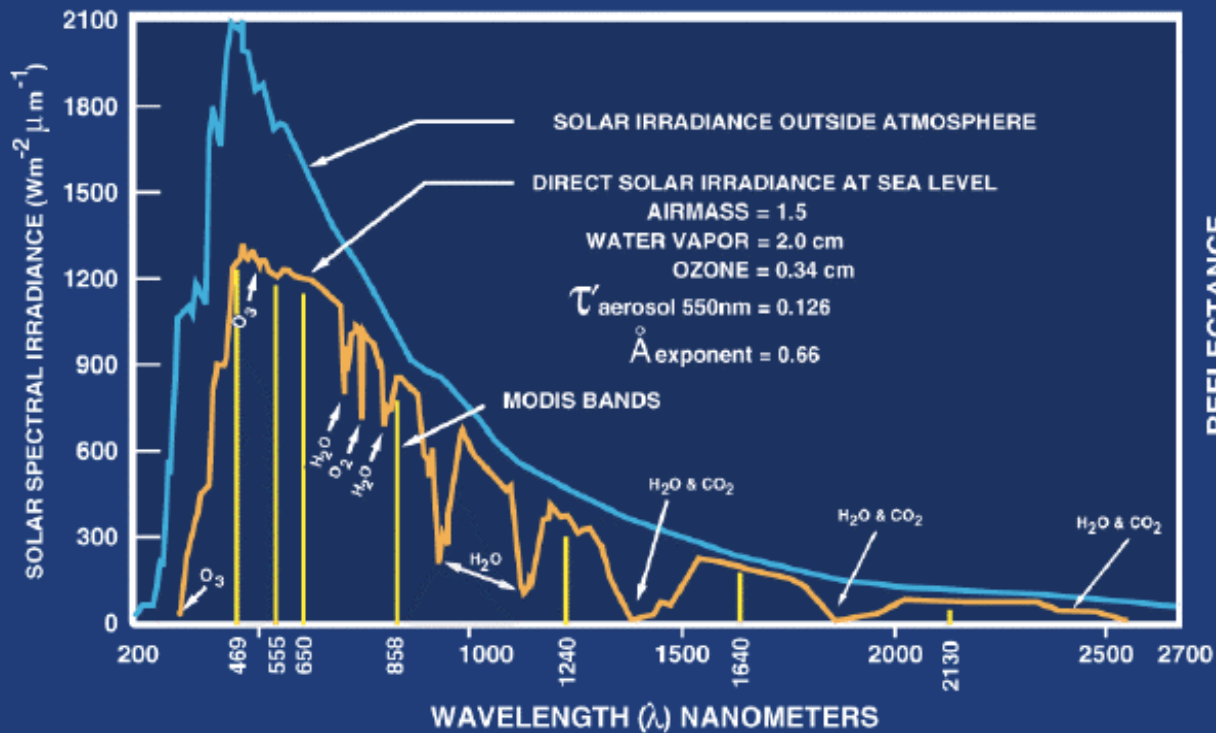


multiple scattering

Atmospheric Correction of Earth  
Observation Data for  
Environmental Monitoring: Theory



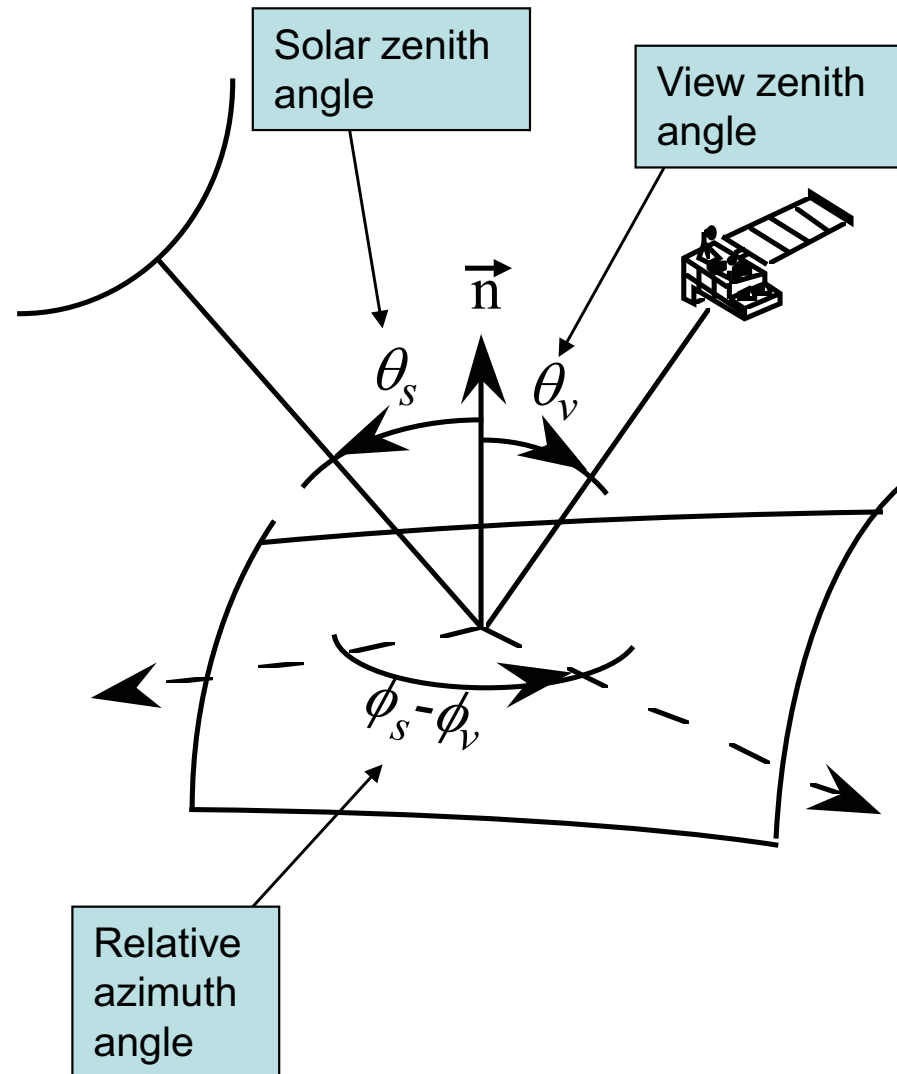
# Solar (reflective) spectral domain




Atmospheric Correction of Earth  
Observation Data for  
Environmental Monitoring Theory



# Observation Geometry





# Solution of the Radiative Transfer in the reflective domain for non absorbing atmosphere and lambertian ground

**Atmospheric reflectance**

**Ground reflectance  
(= albedo for lambertian)**

$$\rho_{app}(\theta_s, \theta_v, \phi) = \rho_{atm}(\theta_s, \theta_v, \phi) + T_{atm}(\theta_s) T_{atm}(\theta_v) \frac{\rho_{ground}}{1 - \rho_{ground} S_{atm}}$$

**Atmospheric  
Transmissions**

**Apparent reflectance at satellite level**

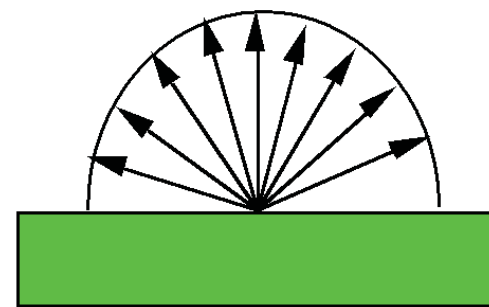
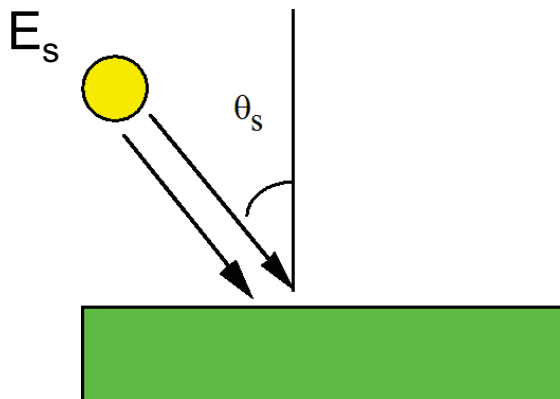
**Atmosphere  
spherical albedo**



# Perfect Lambertian Reflector

Radiance of the Perfect Lambertian Reflector

$$\int_0^\pi \int_0^{2\pi} RPLF(\theta_s, \theta, \phi) \cos(\theta) \sin(\theta) d\theta d\phi = E_s \cos(\theta_s)$$



Isotropic  
radiation

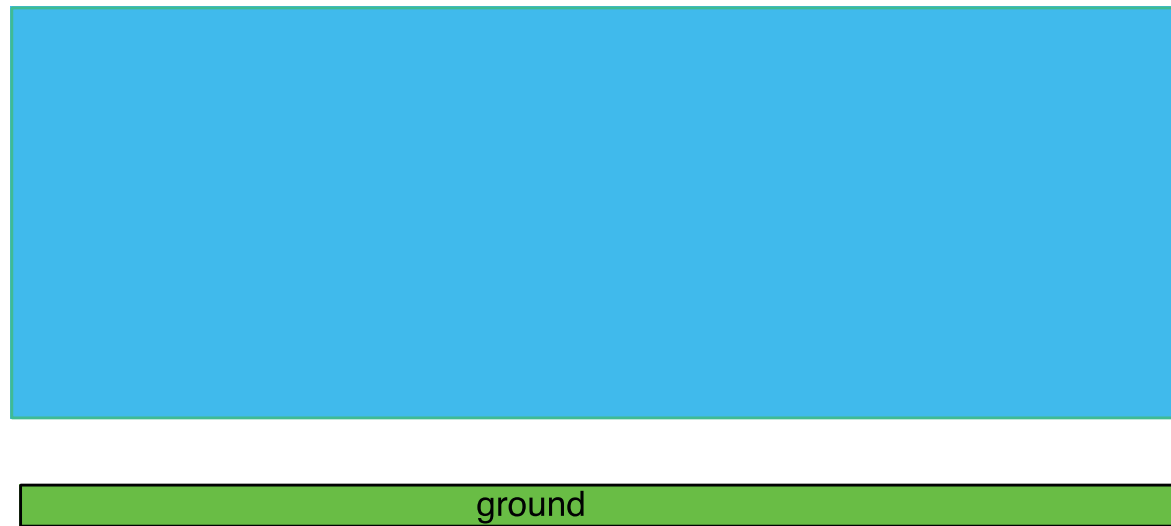
$$\rho_{\text{Perfect Lambertian reflector}}(\theta_s, \theta_v, \phi) = 1$$

$$\rho_{\text{Lambertian reflector}}(\theta_s, \theta_v, \phi) = \rho$$





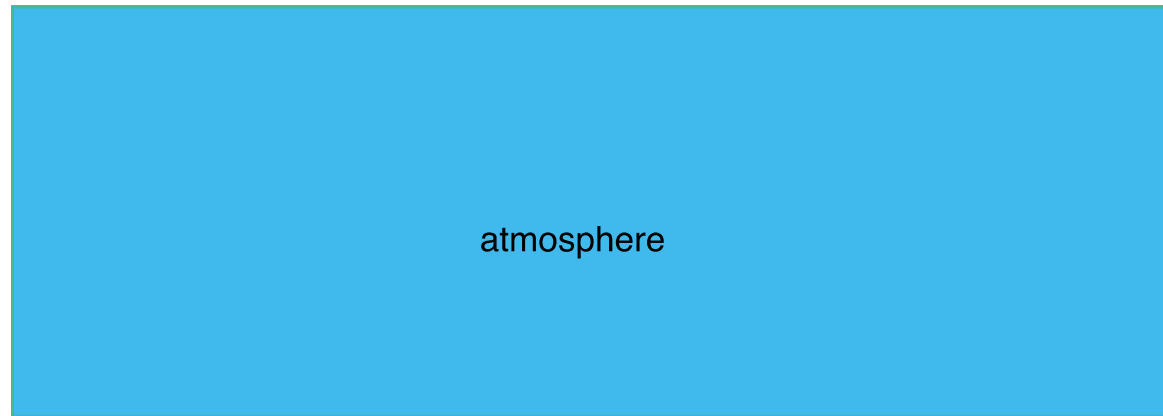
# Simple Radiative Transfer Equation





# SRTE (cont.)

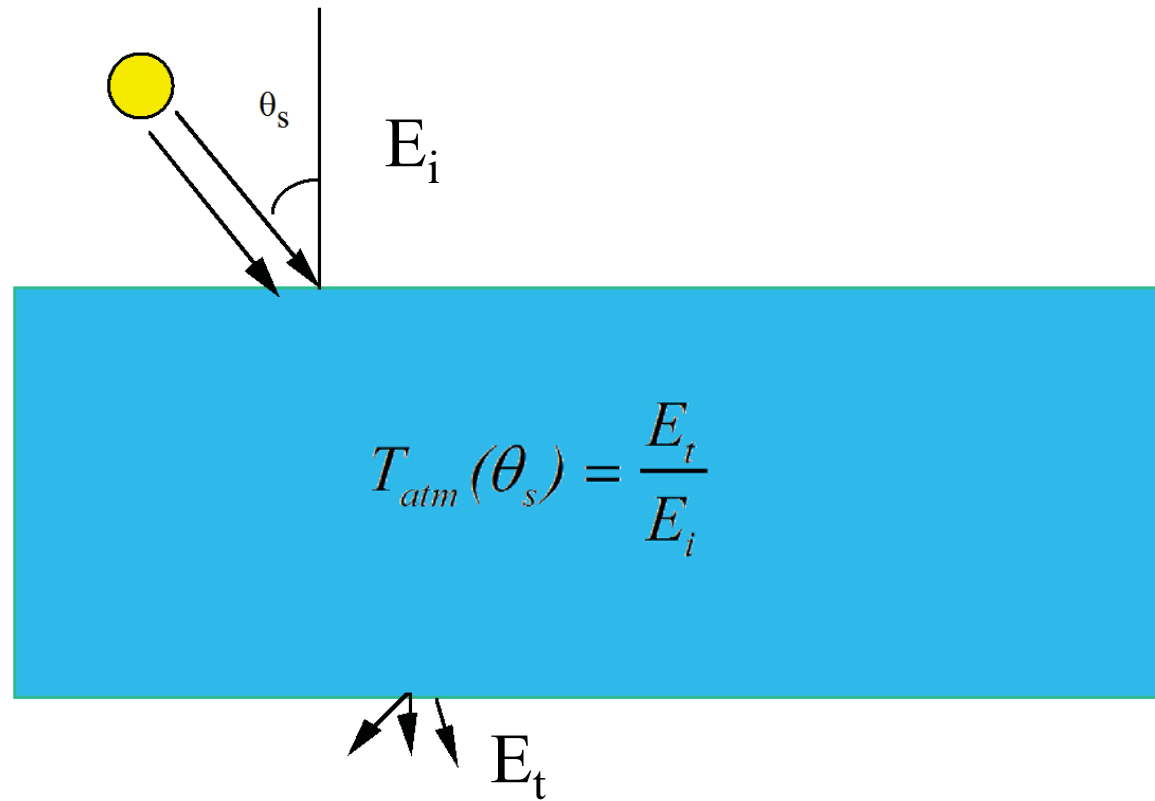
$$\rho_{atm}(\theta_s, \theta_v, \phi)$$



Absorbing ground

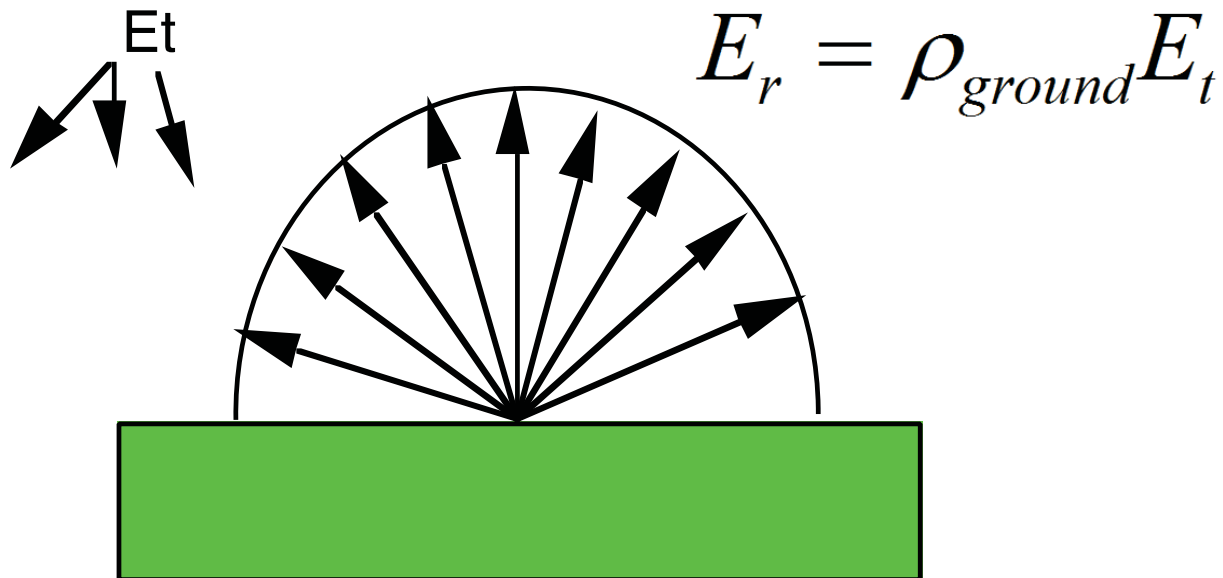


# SRTE (cont.)



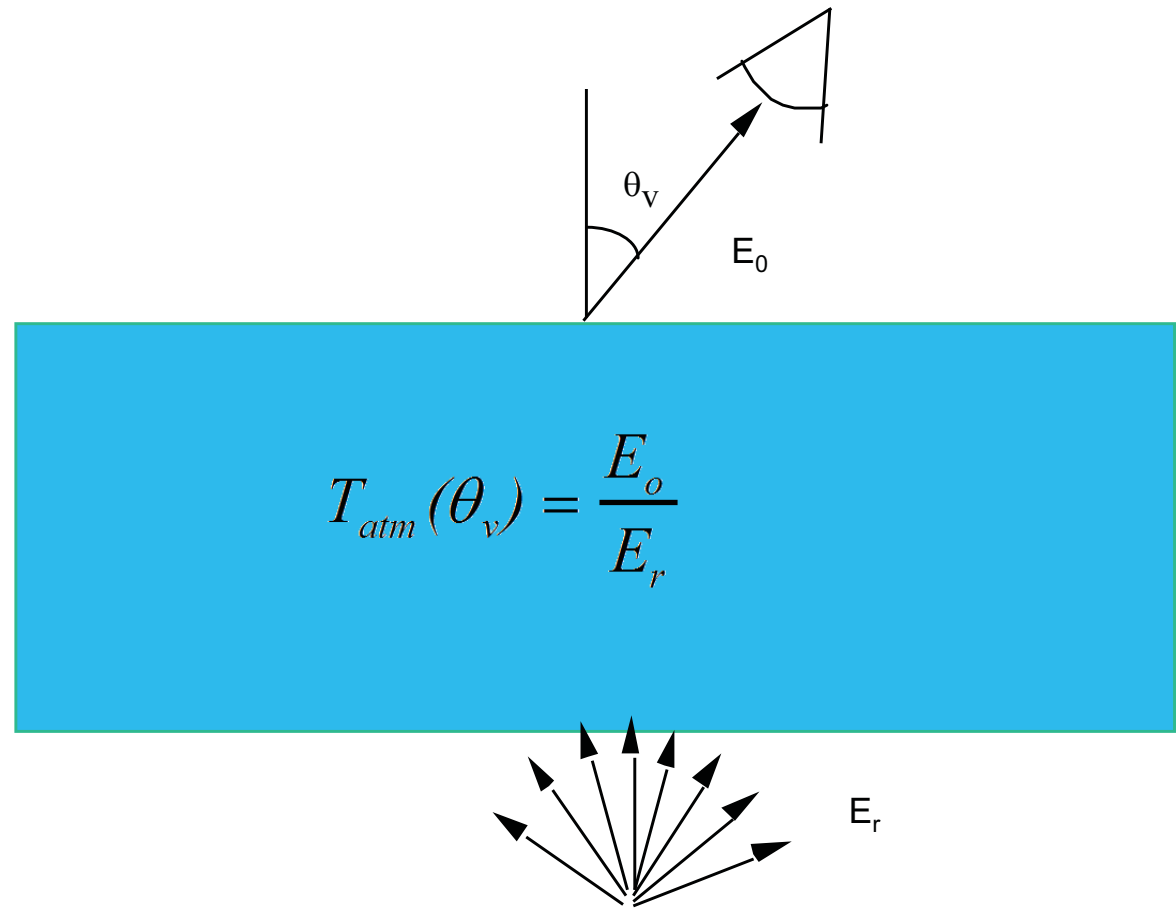


# SRTE (cont.)



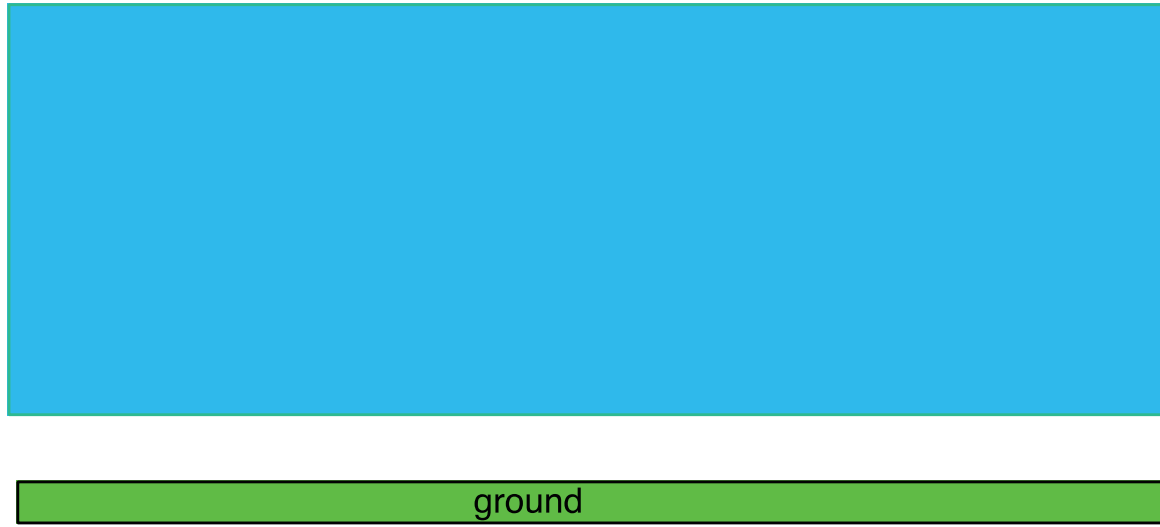


# SRTE (cont.)





# SRTE 1 interaction (cont.)



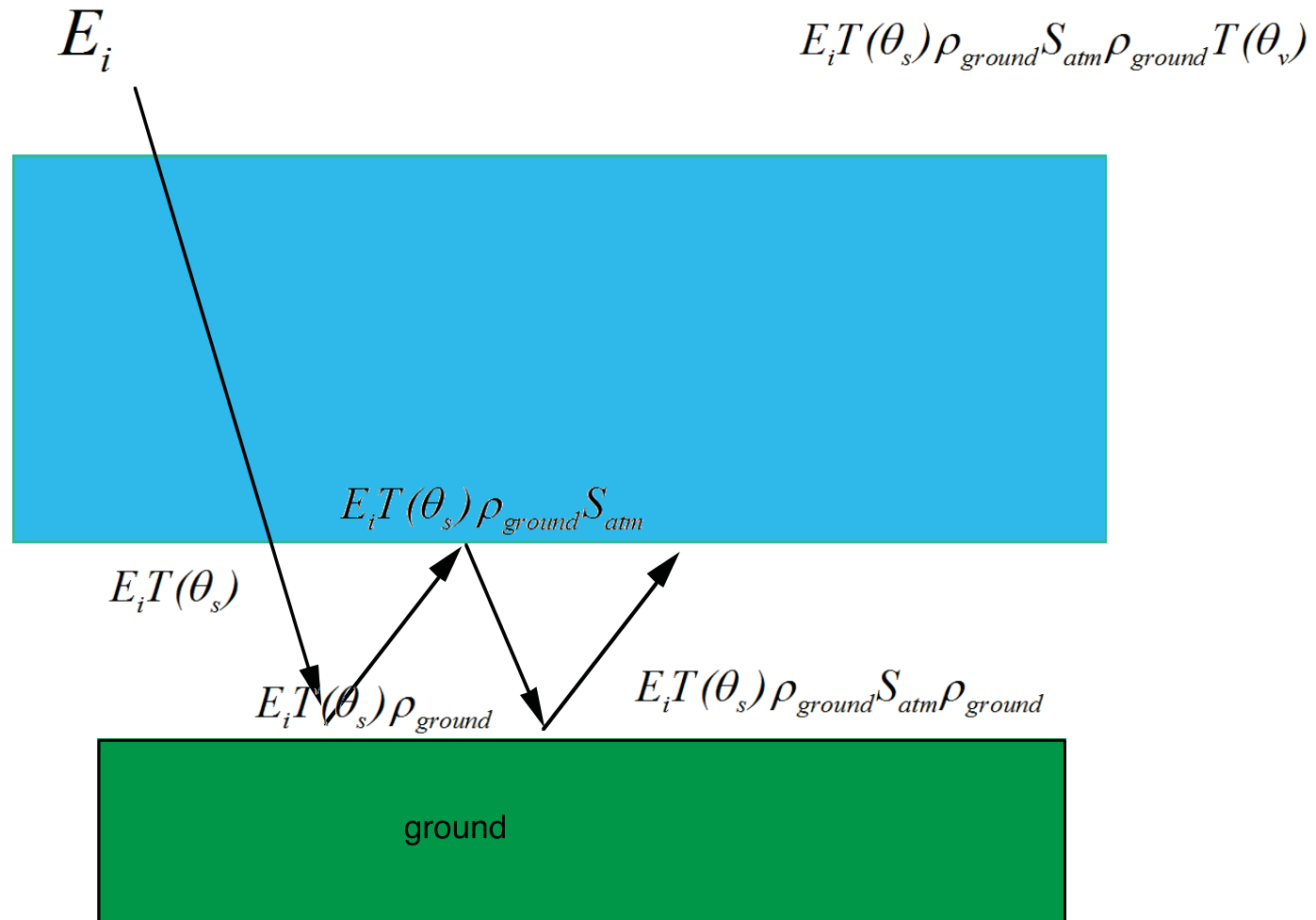
$$\rho_{app} = \rho_{atm} + \frac{E_o}{E_i}$$

$$\frac{E_o}{E_i} = \frac{T(\theta_v)E_r}{E_i} = \frac{T(\theta_v)\rho_{ground}E_t}{E_i} = T(\theta_v)\rho_{ground}T(\theta_s)$$

$$\rho_{app} = \rho_{atm} + T(\theta_v)\rho_{ground}T(\theta_s)$$



# SRTE 2 interactions





# SRTE Multiple Interactions

$$\rho_{app} = \rho_{atm} + T(\theta_s)T(\theta_v)\rho_{ground} \left[ 1 + \rho_{ground}S_{atm} + (\rho_{ground}S_{atm})^2 + (\rho_{ground}S_{atm})^3 \dots \right]$$

$$1 + r + r^2 + r^3 + \dots r^{n-1} = \frac{1 - r^n}{1 - r}$$

$\rho_{ground}S_{atm} < 1$  so when  $n \rightarrow \infty$  then  $(\rho_{ground}S_{atm})^n \rightarrow 0$

**Therefore**  $\left[ 1 + \rho_{ground}S + (\rho_{ground}S)^2 + (\rho_{ground}S)^3 \dots \right] = \frac{1}{1 - \rho_{ground}S}$

$$\rho_{app} = \rho_{atm} + T_{atm}(\theta_s)T_{atm}(\theta_v) \frac{\rho_{ground}}{1 - \rho_{ground}S_{atm}}$$





# STRE for non absorbing atmosphere and lambertian ground

**Atmospheric reflectance**

**Ground reflectance  
(= albedo for lambertian)**

$$\rho_{app}(\theta_s, \theta_v, \phi) = \rho_{atm}(\theta_s, \theta_v, \phi) + T_{atm}(\theta_s) T_{atm}(\theta_v) \frac{\rho_{ground}}{1 - \rho_{ground} S_{atm}}$$

**Atmospheric  
Transmissions**

**Apparent reflectance at satellite level**

**Atmosphere  
spherical albedo**

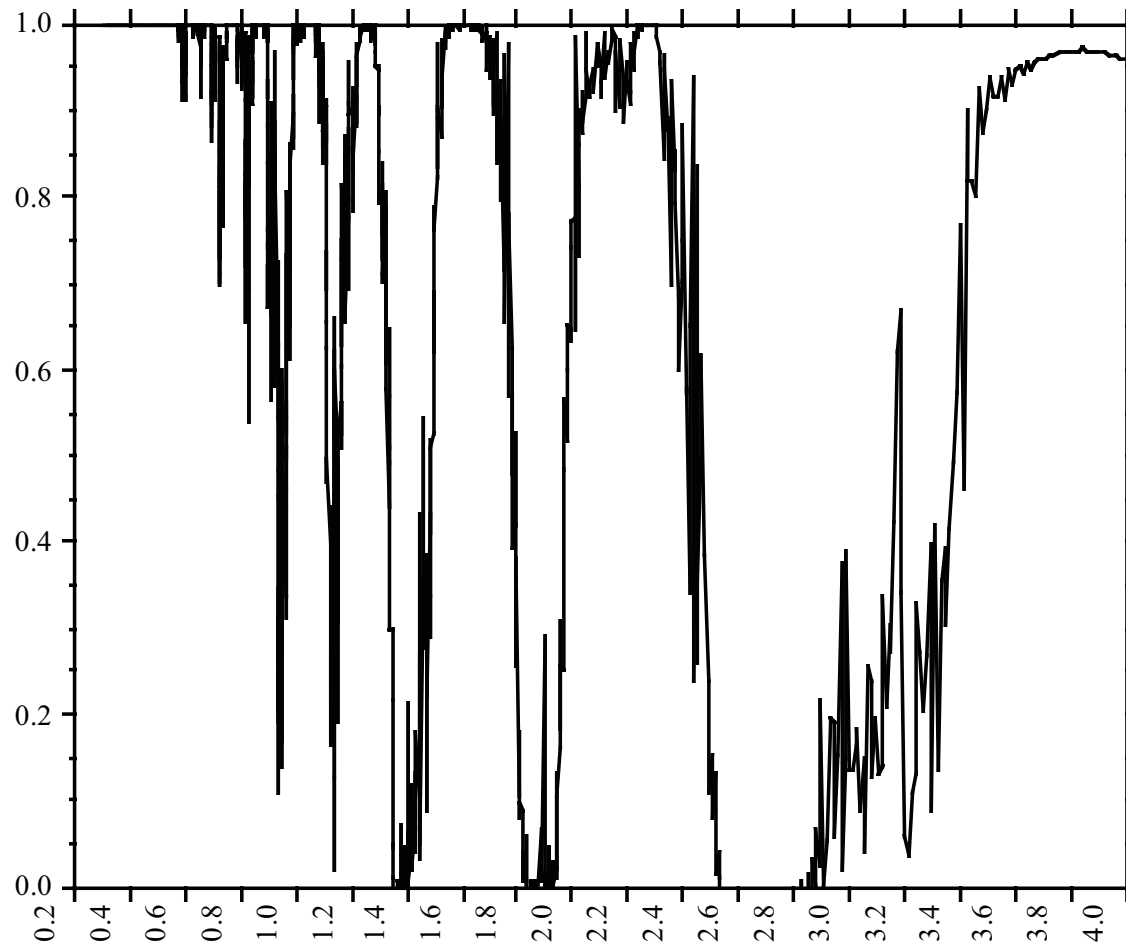


# The composition of the atmosphere

Permanent Constituents		Variable constituents	
Constituent	% by volume	Constituent	% by volume
Nitrogen (N <sub>2</sub> )	78.084	Water Vapor (H <sub>2</sub> O)	0.04
Oxygen (O <sub>2</sub> )	20.948	Ozone (O <sub>3</sub> )	12 x 10 <sup>-4</sup>
Argon (Ar)	0.934	Sulfur dioxide (SO <sub>2</sub> ) <sup>b</sup>	0.001 x 10 <sup>-4</sup>
Carbon dioxide (CO <sub>2</sub> )	0.033	Nitrogen dioxide (NO <sub>2</sub> )	0.001 x 10 <sup>-4</sup>
Neon (Ne)	18.18 x 10 <sup>-4</sup>	Ammonia (NH <sub>3</sub> )	0.001 x 10 <sup>-4</sup>
Helium (He)	5.24 x 10 <sup>-4</sup>	Nitric oxide (NO)	0.0005 x 10 <sup>-4</sup>
Krypton (Kr)	1.14 x 10 <sup>-4</sup>	Hydrogen sulfide (H <sub>2</sub> S)	0.00005 x 10 <sup>-4</sup>
Xenon (Xe)	0.089 x 10 <sup>-4</sup>	Nitric acid vapor	trace
Hydrogen (H <sub>2</sub> )	0.5 x 10 <sup>-4</sup>		
Methane (CH <sub>4</sub> )	1.5 x 10 <sup>-4</sup>		
Nitrous Oxide (N <sub>2</sub> O)	0.27 x 10 <sup>-4</sup>		
Carbon Monoxide (CO)	0.19 x 10 <sup>-4</sup>		



# Gaseous Absorption ( $\text{H}_2\text{O}$ )





# Modified SRTM to account for absorption

In case of a pure molecular atmosphere (no aerosol) we can write:

$$\rho_{app}(\theta_s, \theta_v, \phi) = Tg^{othergases}(m, U_{gaz}) \left[ \rho_{atm}(\theta_s, \theta_v, \phi) + Tg^{H_2O}(m, U_{H_2O}) T_{atm}(\theta_s) T_{atm}(\theta_v) \frac{\rho_{ground}}{1 - S_{atm} \rho_{ground}} \right]$$

$m$  is the air mass =  $1/\cos(\theta_s) + 1/\cos(\theta_v)$

$U_{gaz}$  is the gaz concentration



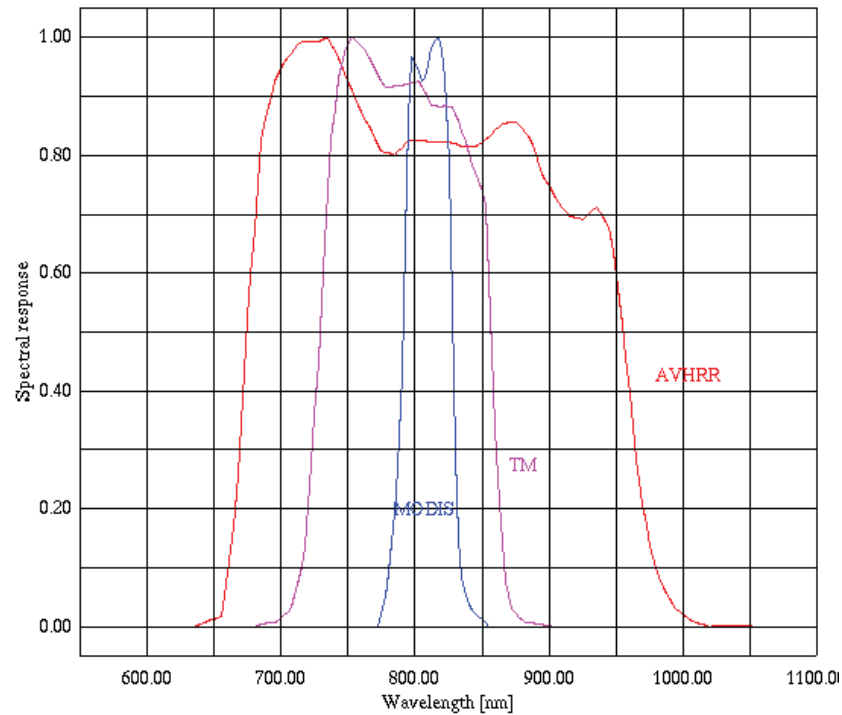
# Final SRTTE approximation

$$\rho_{app}(\theta_s, \theta_v, \phi) \sim Tg^{othergases}(m, U_{gaz}) \left[ \begin{aligned} &\rho_R(\theta_s, \theta_v, \phi) + Tg^{H_2O}(m, U_{H_2O} / 2) \rho_A(\theta_s, \theta_v, \phi) \\ &+ Tg^{H_2O}(m, U_{H_2O}) T_A(\theta_s) T_A(\theta_v) T_R(\theta_s) T_R(\theta_v) \frac{\rho_{ground}}{1 - S_{R+A} \rho_{ground}} \end{aligned} \right]$$

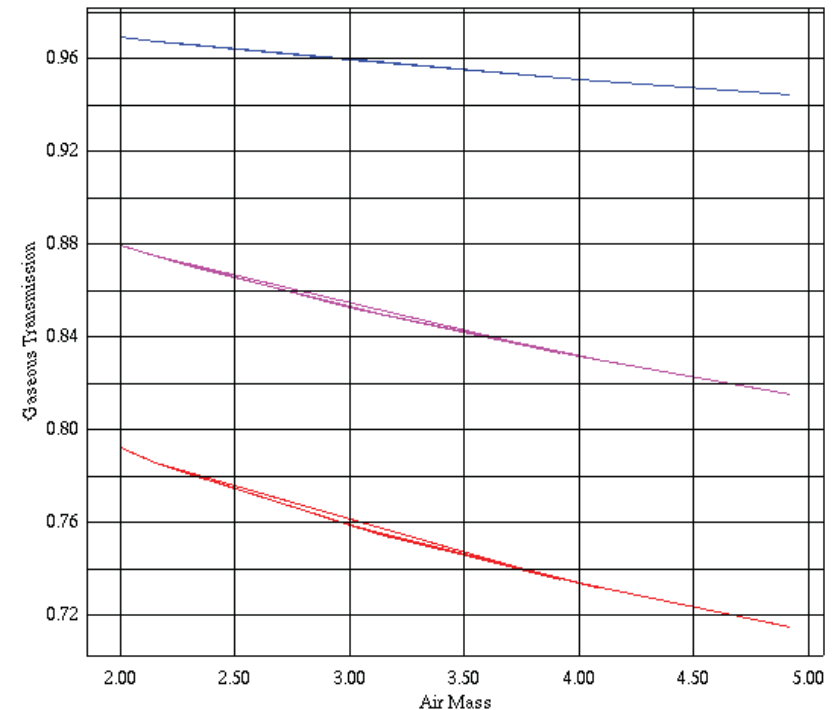


# Water vapor effect for different sensors in the near infrared

Spectral response of the near infrared band



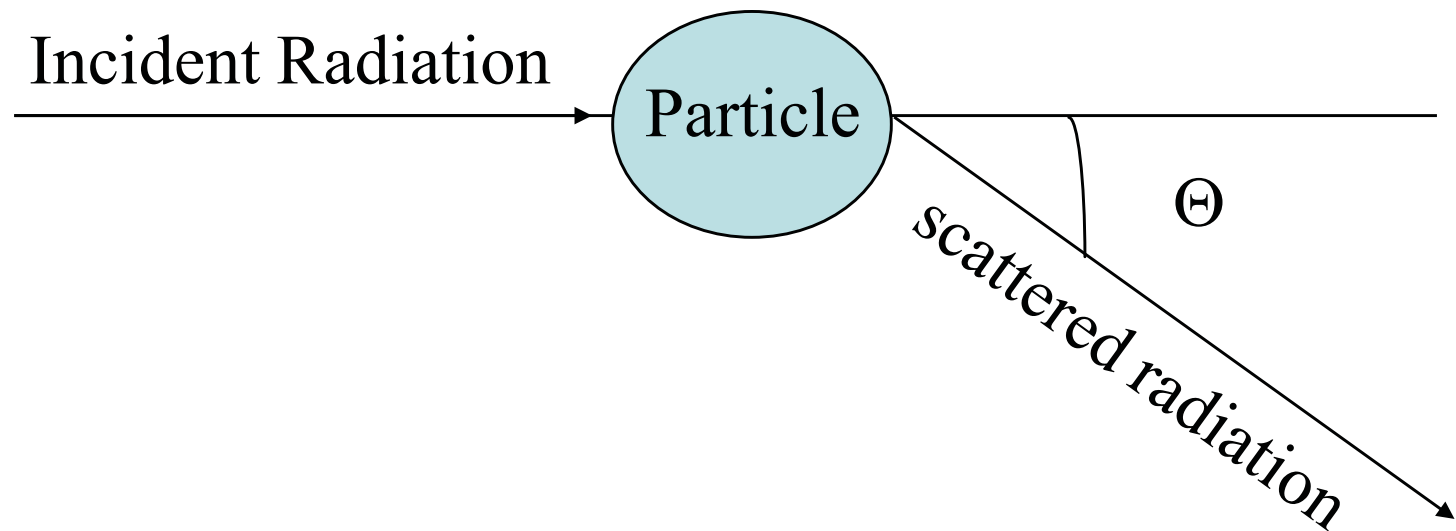
Gaseous transmission: Water vapor, Tropical atmosphere





# Scattering angle ,

- The scattering angle,  $\Theta$ , is the relative angle between the incident and the scattered radiation





# Phase function

- The phase function,  $P(\Theta)$ , describe the distribution of scattered radiation for one or an set of particles. It is normalized such as:

$$\int_0^{2\pi} \int_0^{\pi} P(\Theta) d\omega = 4\pi$$

since

$$\int_0^{2\pi} \int_0^{\pi} P(\Theta) \sin(\theta) d\theta d\phi = 2\pi \int_0^{\pi} P(\theta) \sin(\theta) d\theta$$

we have

$$\int_0^{\pi} P(\theta) \sin(\theta) d\theta = 2$$



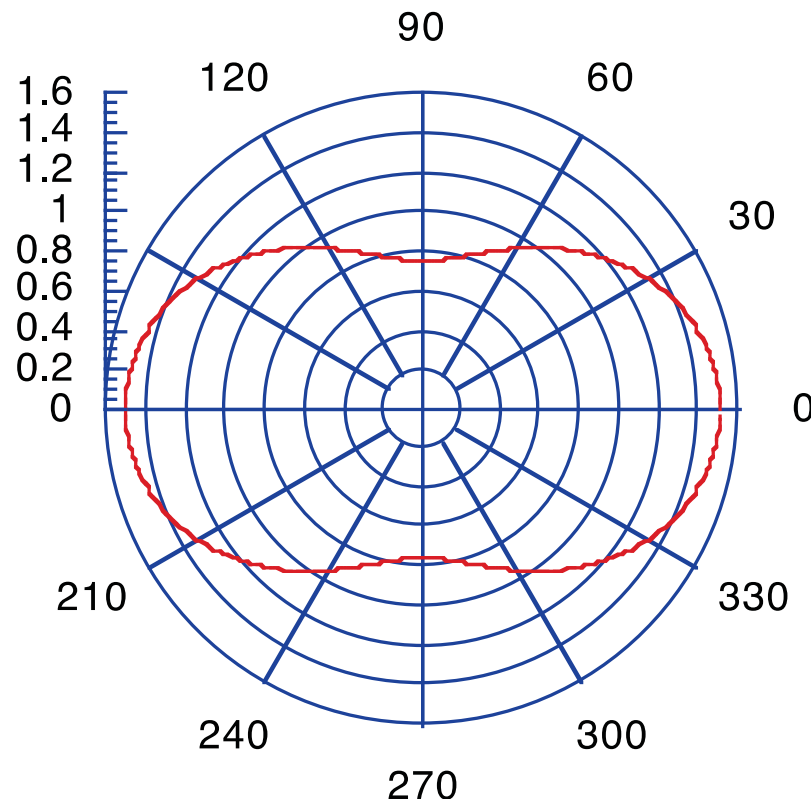


# Rayleigh/molecular scattering

## 1/4

- Rayleigh or molecular scattering refers to scattering by atmospheric gases, in that case:

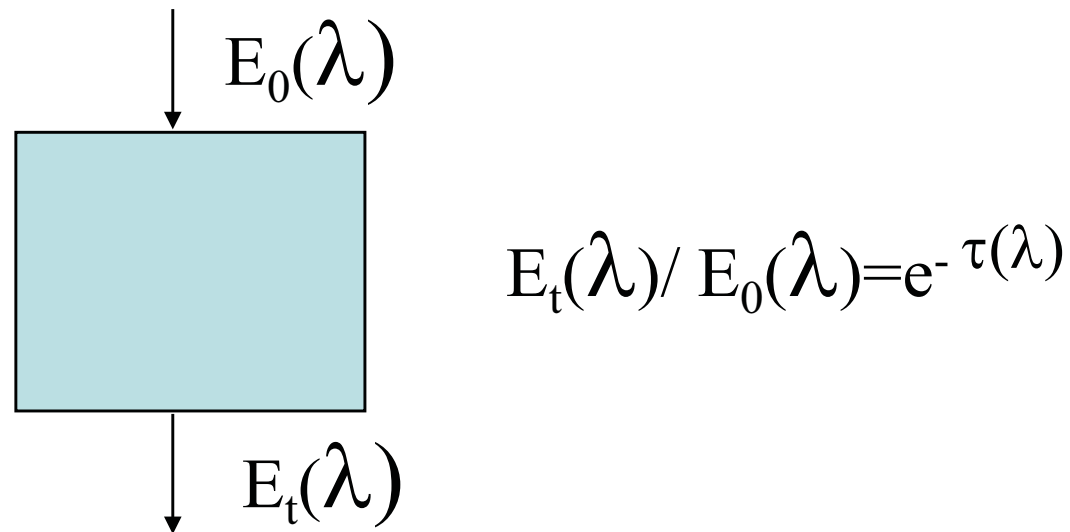
$$P(\Theta) = \frac{3}{4} (1 + \cos^2(\Theta))$$





# Rayleigh/molecular scattering 2/4

- The concentration in scatterer is better described by the efficiency they scatter at a certain wavelength or the proportion of direct transmission which is related to the spectral optical thickness  $\tau(\lambda)$



- For Rayleigh  $\tau(\lambda)$  is proportional to  $\lambda^{-4}$  and for standard pressure is  $\sim 0.235$  at  $0.45 \mu\text{m}$



# Rayleigh/molecular scattering

3/4

- The rayleigh reflectance,  $\rho_R$ , could be crudely approximated by:

$$\rho_R^\lambda(\theta_s, \theta_v, \phi) \sim \frac{\tau_R^\lambda P(\Theta)}{4 \cos(\theta_s) \cos(\theta_v)}$$



## Rayleigh/molecular scattering 4/4

- Compute the reflectance of the sky (assumed clear no aerosol) at solar noon at 45degree latitude at vernal equinox looking straight up at  $0.45\mu\text{m}$ ,  $0.55\mu\text{m}$ ,  $0.65\mu\text{m}$

$$\rho_R^{0.45\mu\text{m}}(45^\circ, 0^\circ, \phi) = \frac{0.235P(45^\circ)}{4\cos(45^\circ)\cos(0^\circ)}$$
$$= \frac{0.235 \times 0.75(1 + \cos^2(45))}{4\cos(45^\circ)\cos(0^\circ)} = \frac{0.265}{2.8} \sim 0.1$$

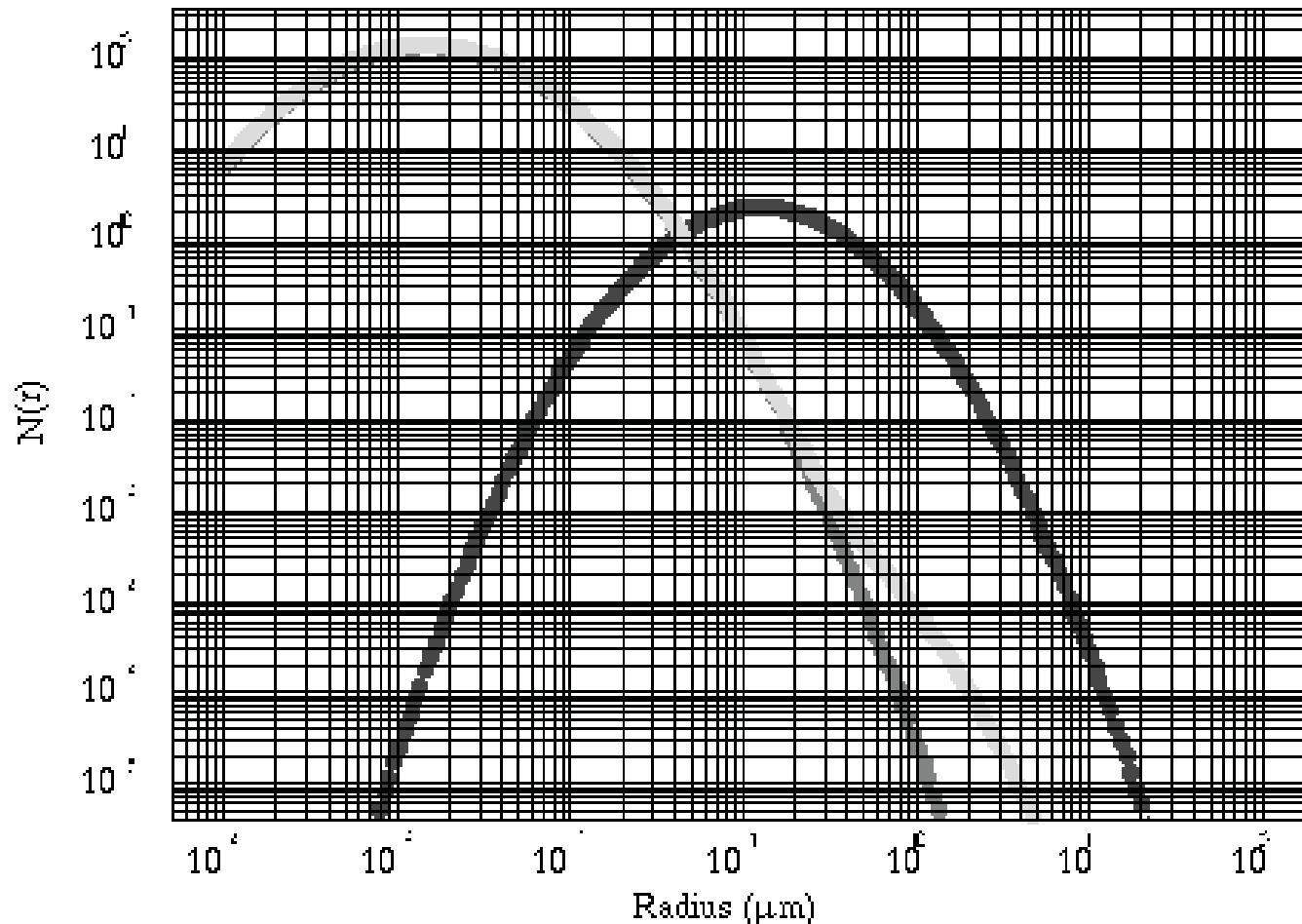
$$\rho_R^{0.55\mu\text{m}} = 0.1 \times (0.45 / 0.55)^4 \sim 0.1 \times 0.4 = 0.04$$

$$\rho_R^{0.65\mu\text{m}} = 0.04 \times (0.55 / 0.65)^4 \sim 0.04 \times 0.5 = 0.02$$



# Aerosol scattering 1/5

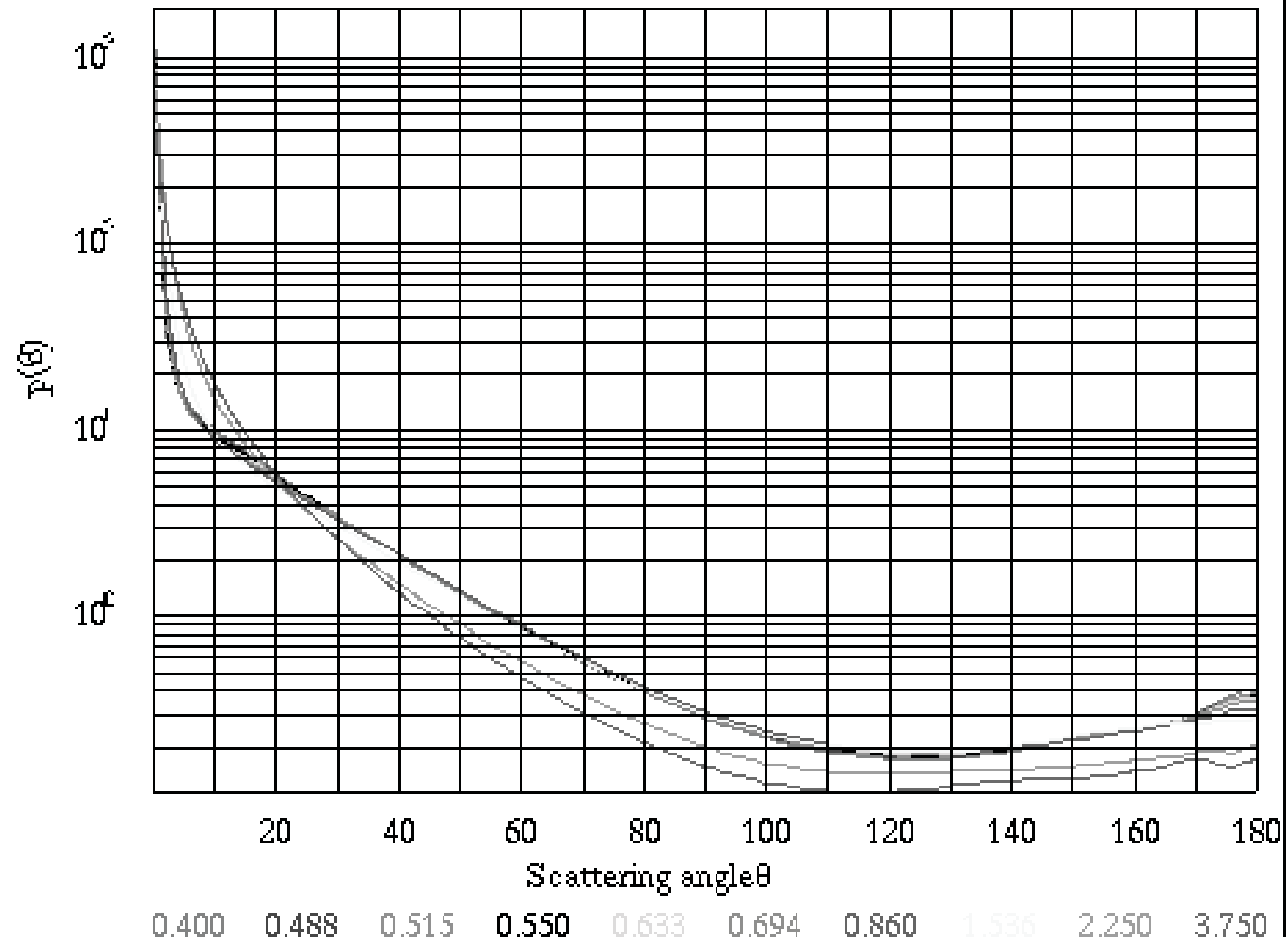
- aerosol scattering refers to scattering by particles in suspension in the atmosphere (not molecules). The MIE scattering theory could be applied to compute the aerosol phase function and spectral optical depth, based on size distribution, real and imaginary index.





# Aerosol scattering 2/5

## Continental aerosol phase function





# Aerosol scattering 3/5

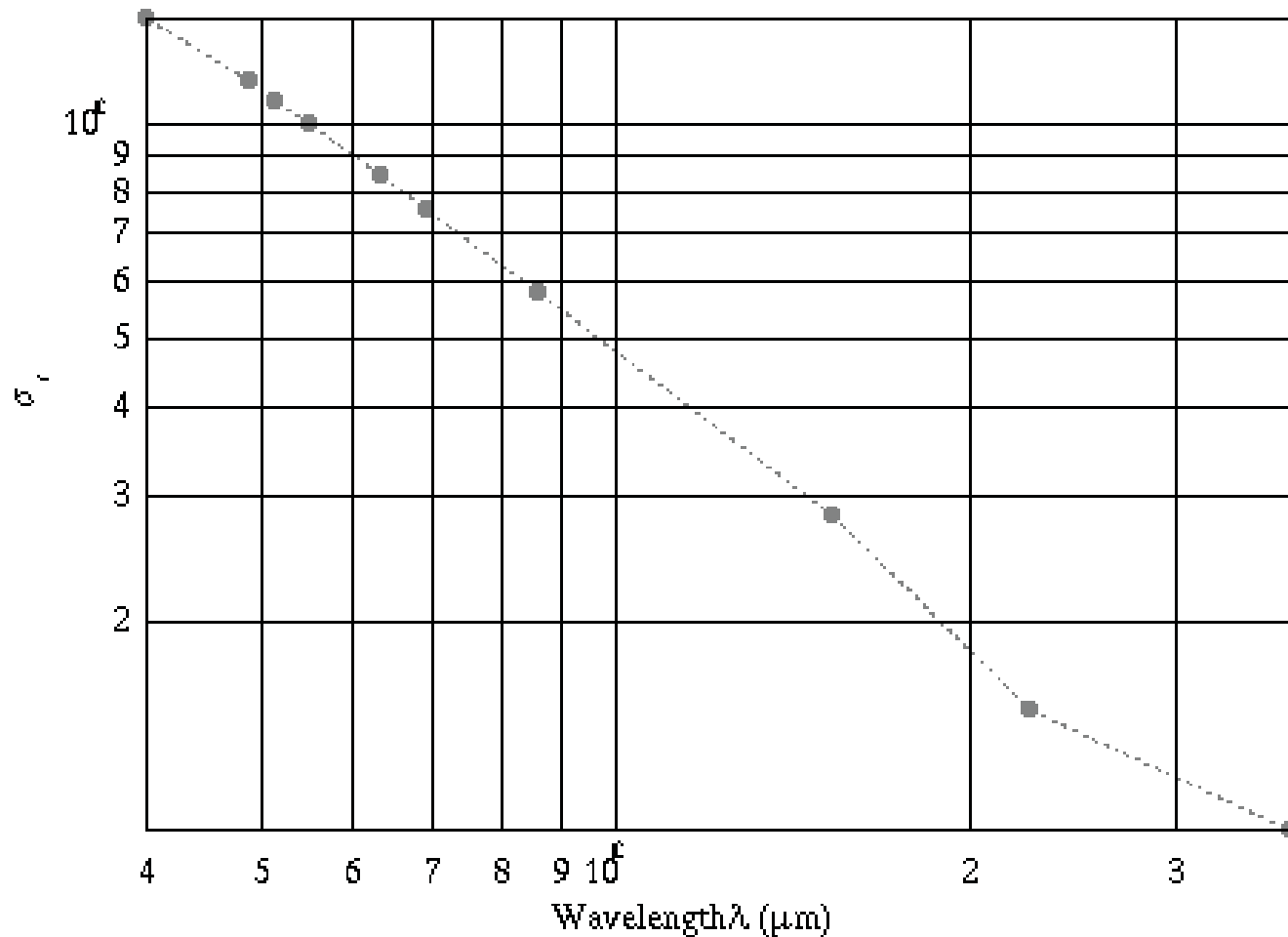
single scattering albedo  
(0.2-1.0) to account for absorbing  
particles

$$\rho_A^\lambda(\theta_s, \theta_v, \phi) \sim \frac{\omega_A^\lambda \tau_A^\lambda P(\Theta)}{4 \cos(\theta_s) \cos(\theta_v)}$$



# Aerosol scattering 4/5

Continental aerosol optical thickness spectral variation

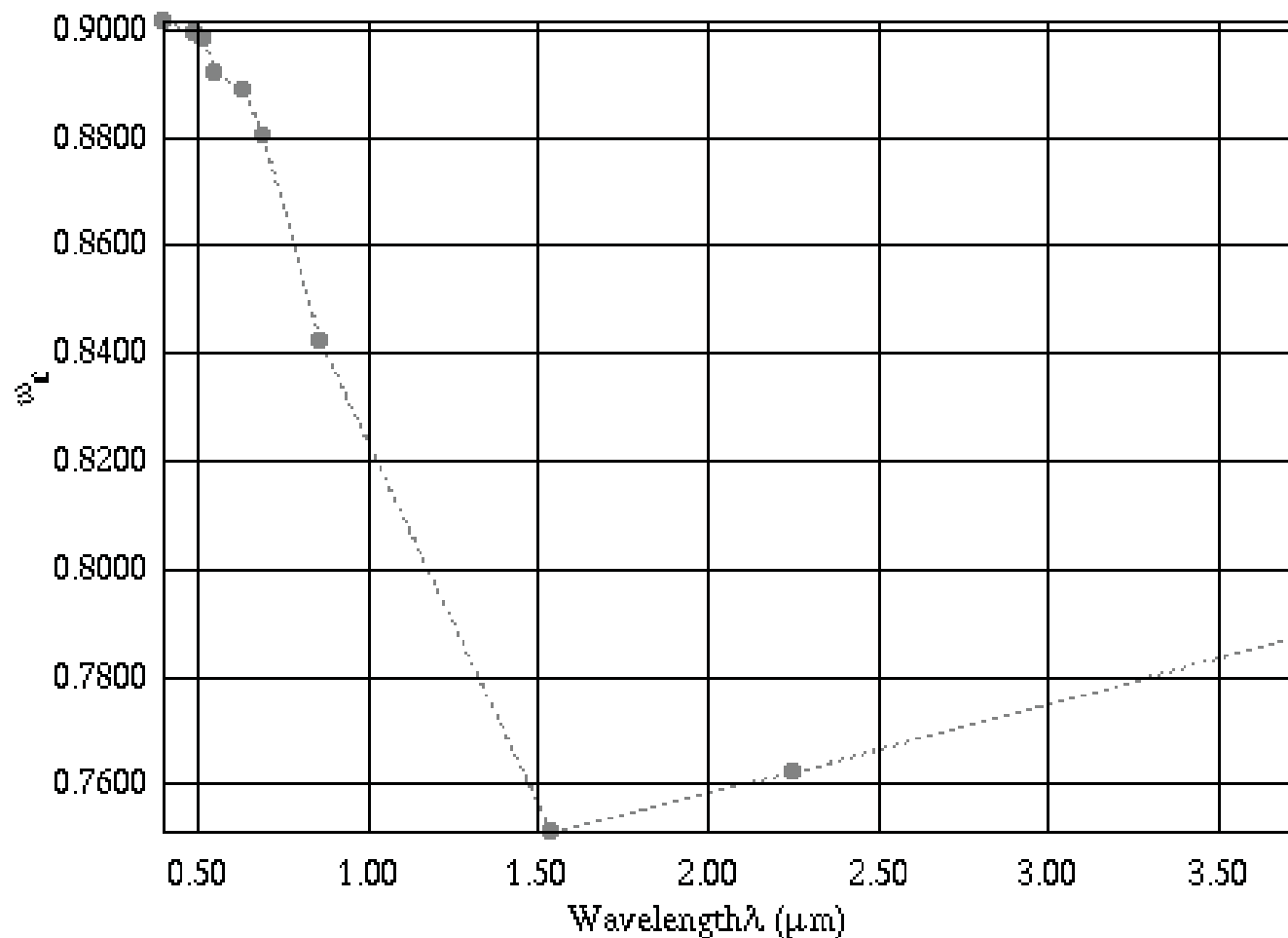






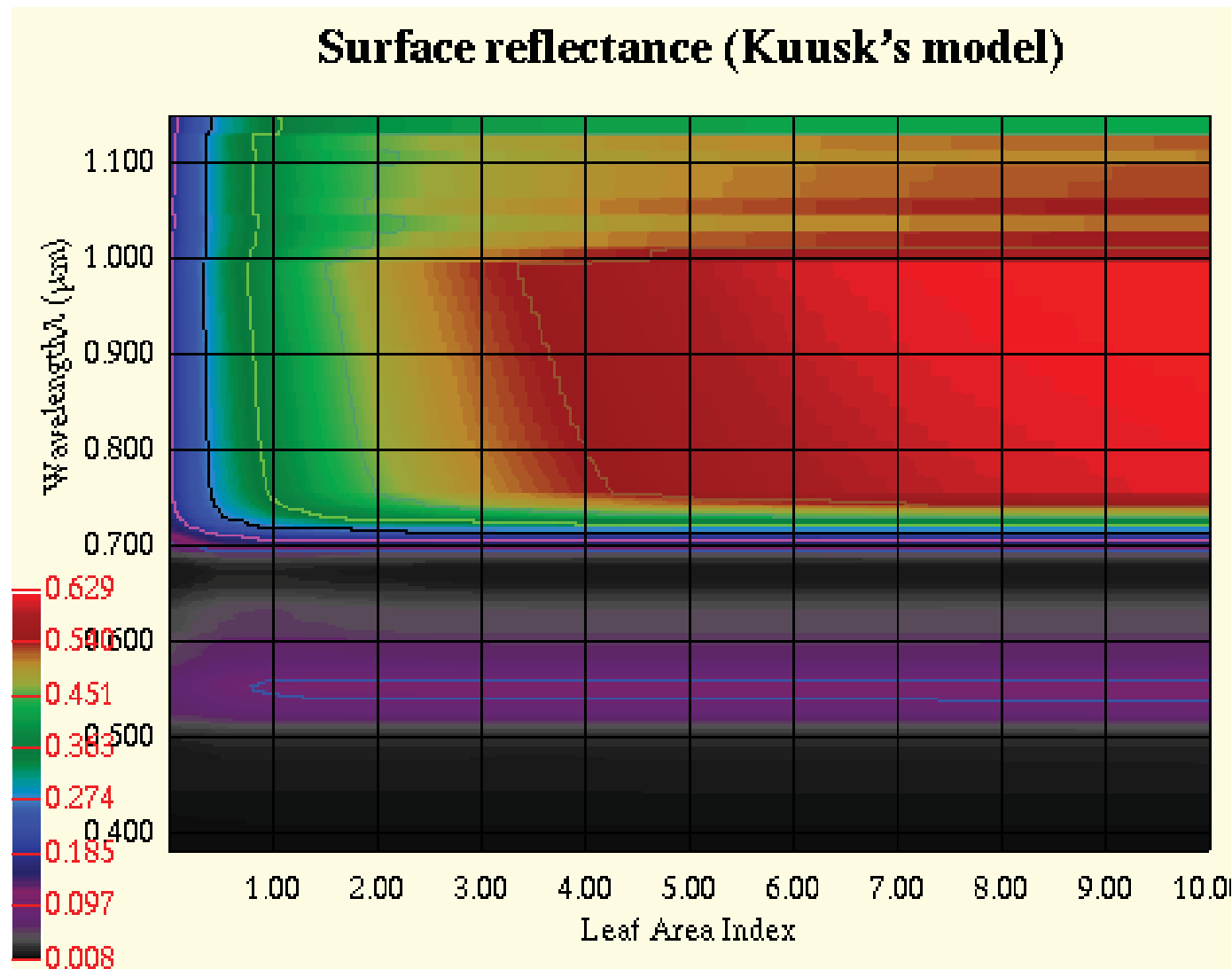
# Aerosol scattering 5/5

Continental aerosol single scattering albedo spectral variation



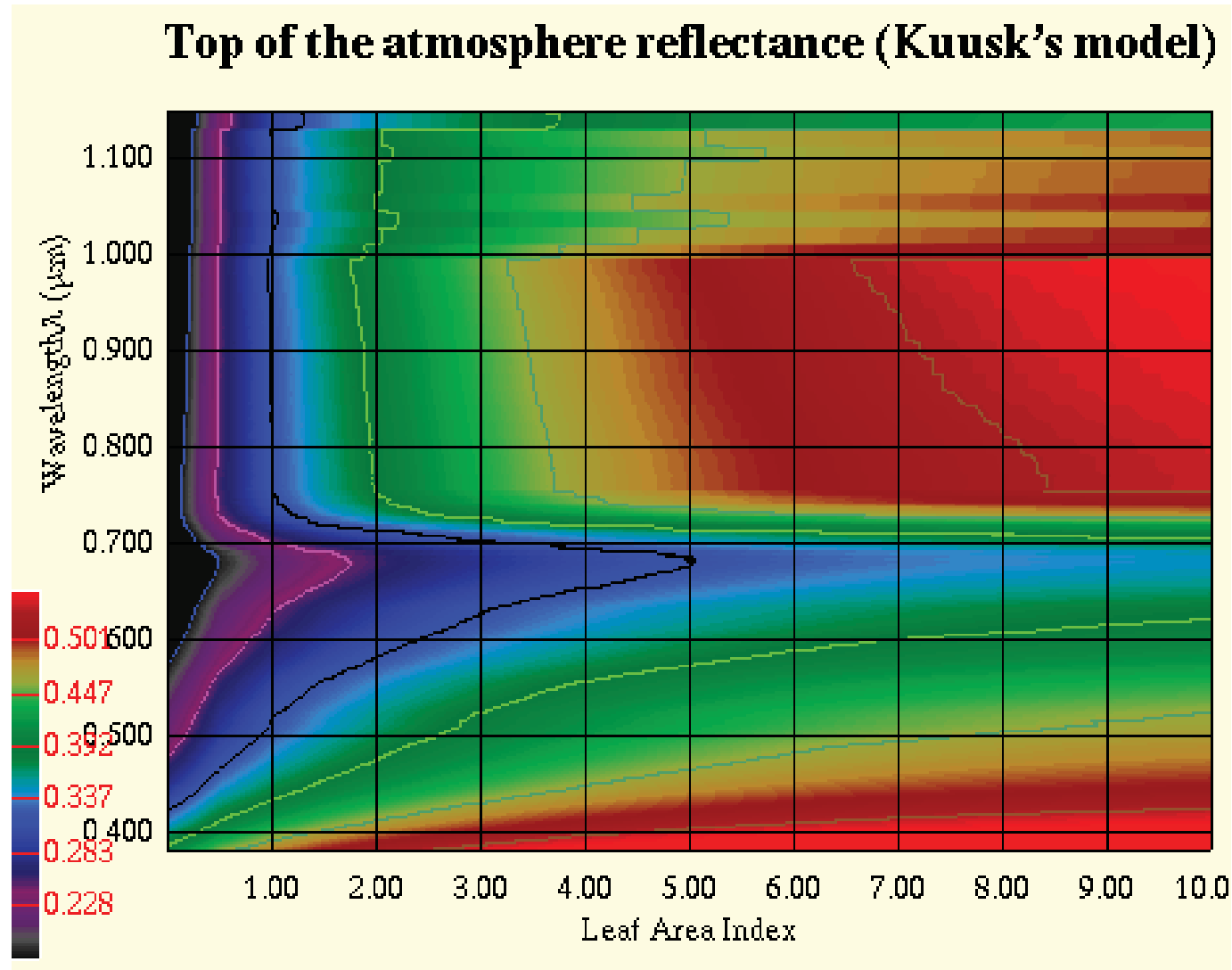


# Atmospheric effect: Vegetation 1/3





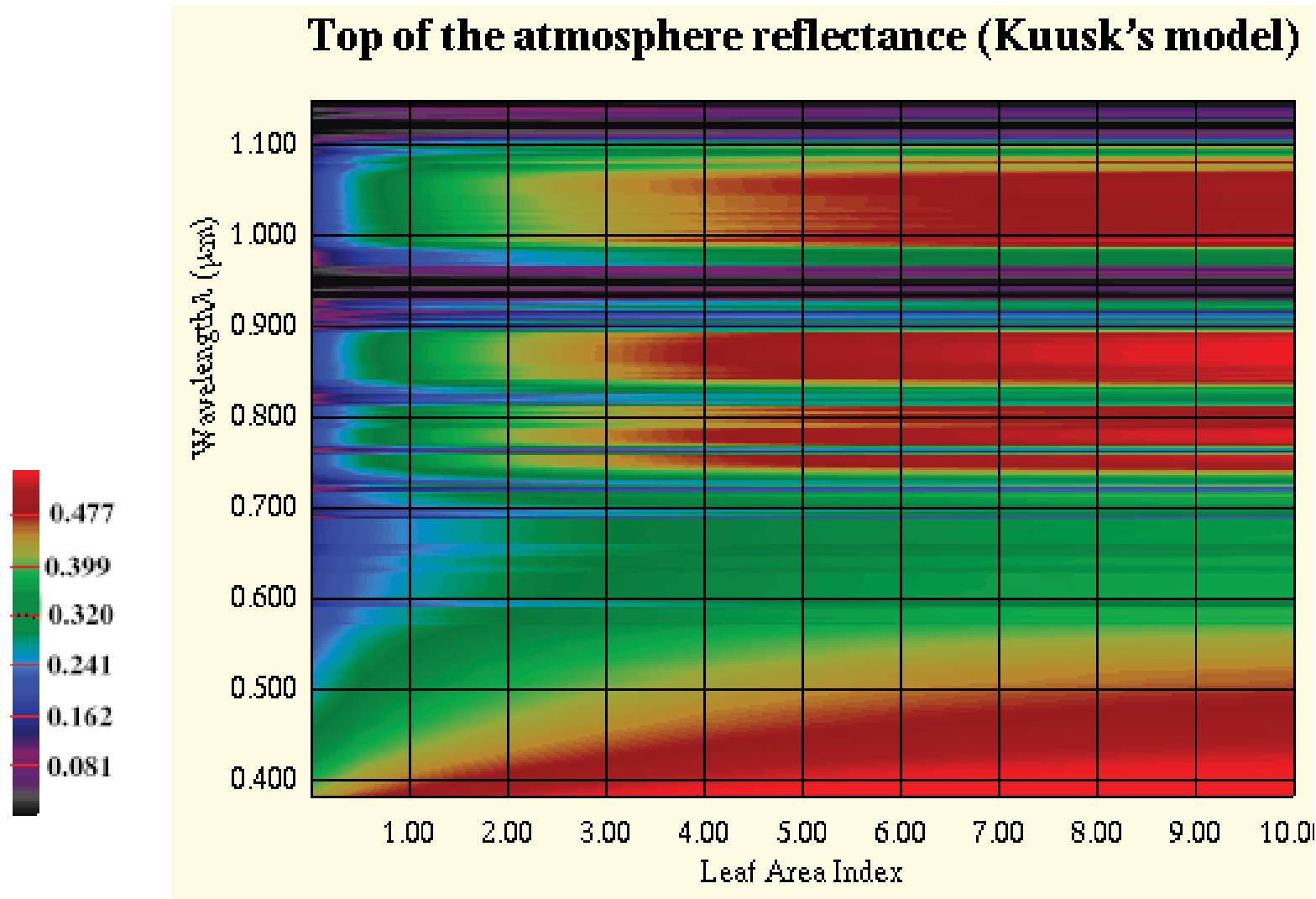
# Atmospheric effect: Vegetation 2/3



No absorption, Continental aerosol



# Atmospheric effect: Vegetation 3/3

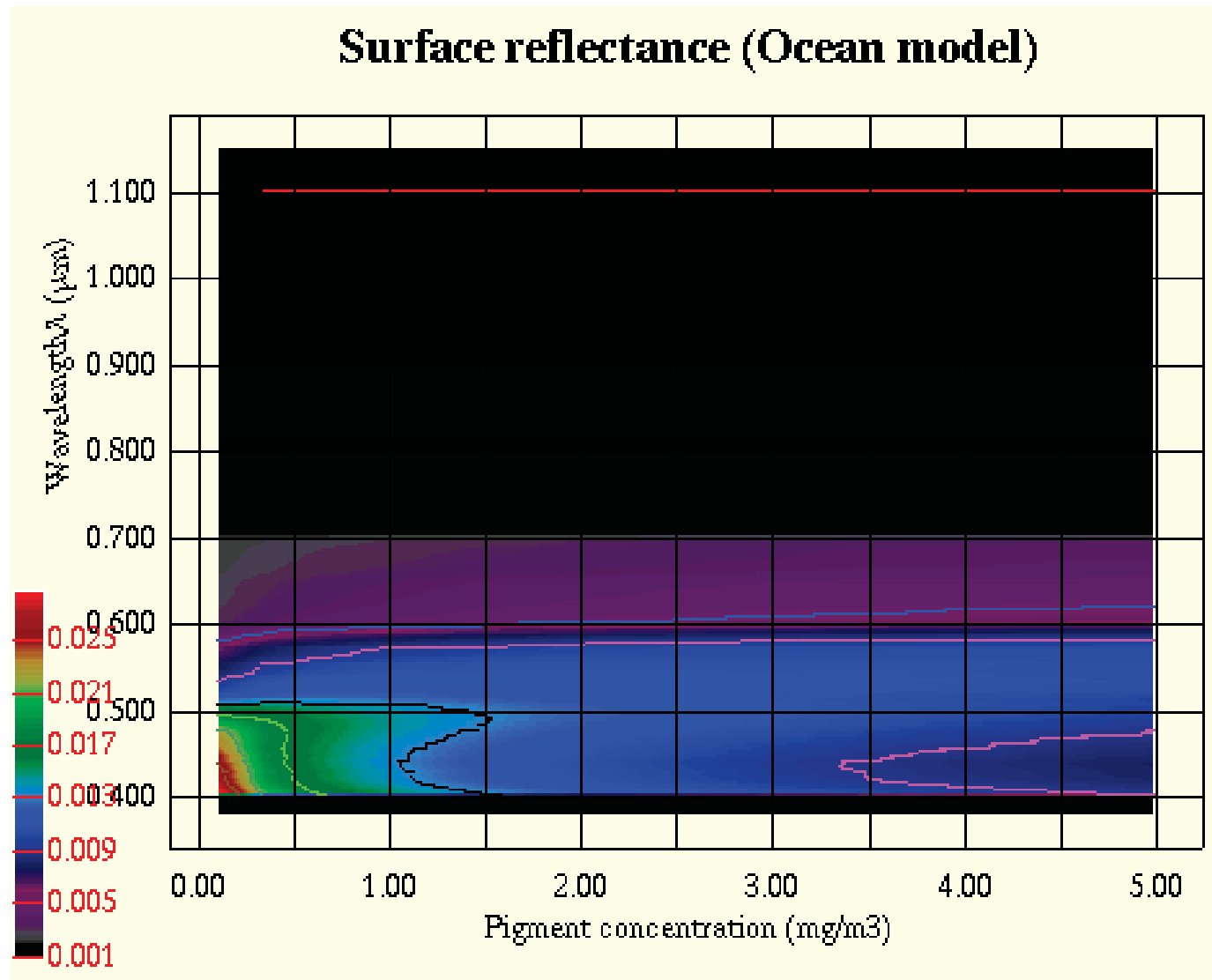


Absorption tropical atmosphere, Continental aerosol

Atmospheric Correction of Earth Observation Data for Environmental Monitoring: Theory and Best Practises

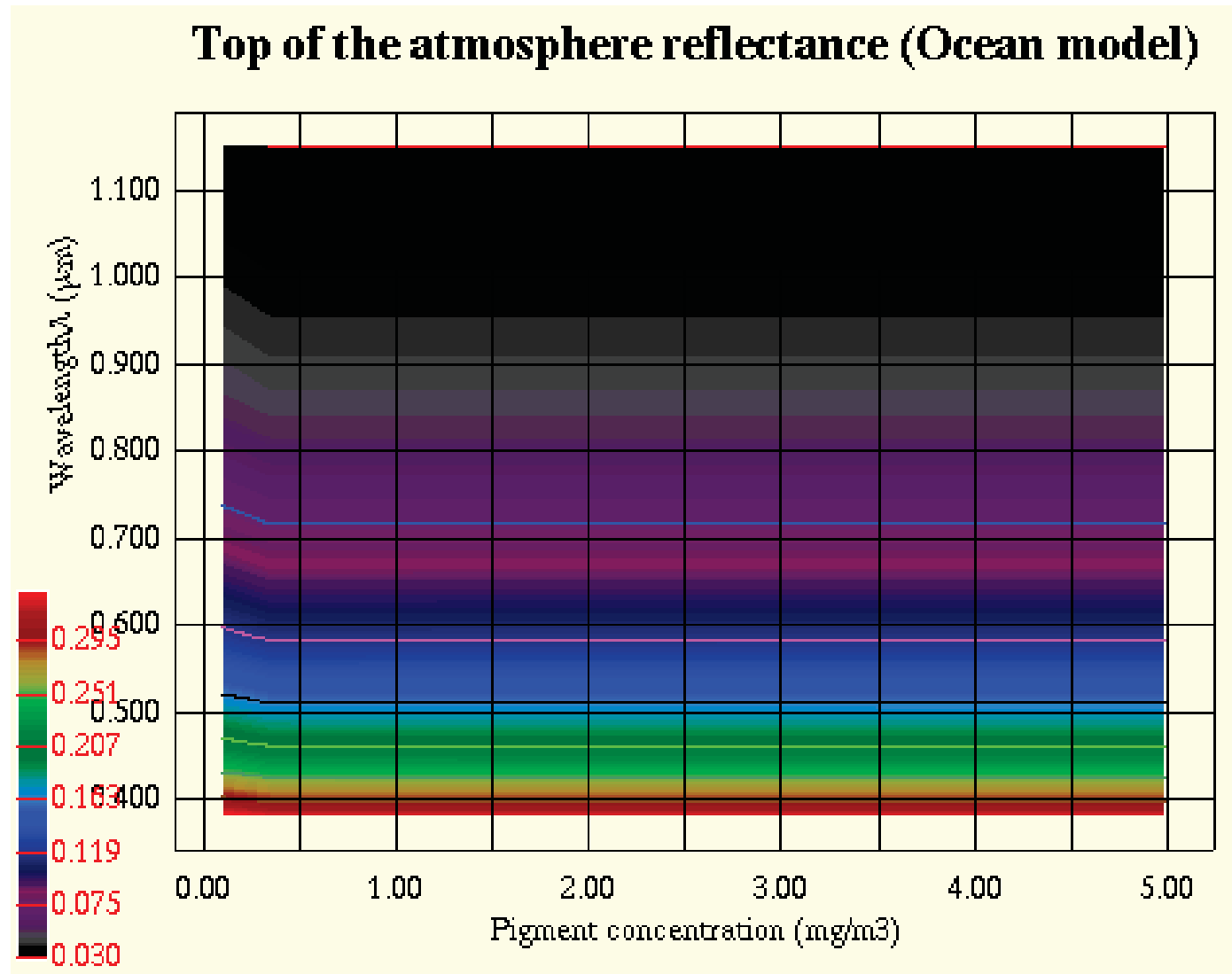


# Atmospheric effect: Ocean 1/2





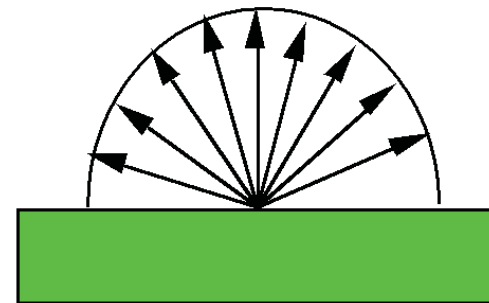
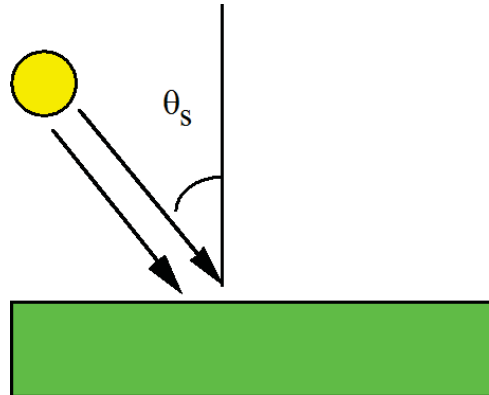
# Atmospheric effect: Ocean 2/2





# Perfect Lambertian Reflector

$$\int_0^{\pi} \int_0^{2\pi} RPLF(\theta_s, \theta, \phi) \cos(\theta) \sin(\theta) d\theta d\phi = E_s \cos(\theta_s)$$

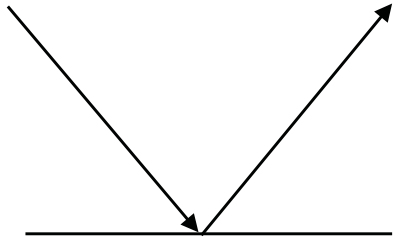


Isotropic  
radiation

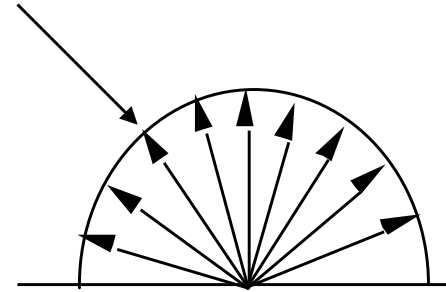
$$RPLF(\theta_s, \theta_v, \phi) = \frac{E_s \cos(\theta_s)}{\pi}$$



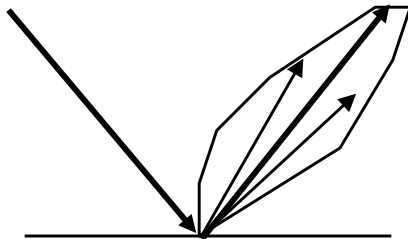
# Different Types of Reflectors



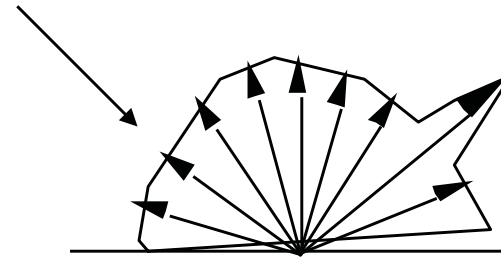
Specular reflector (mirror)



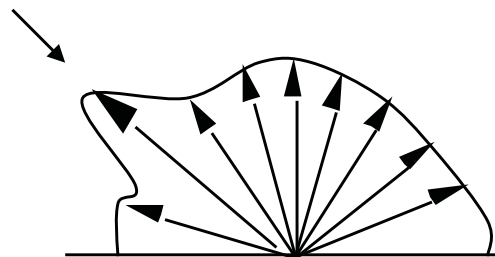
diffuse reflector (Lambertian)



Nearly Specular reflector (water)



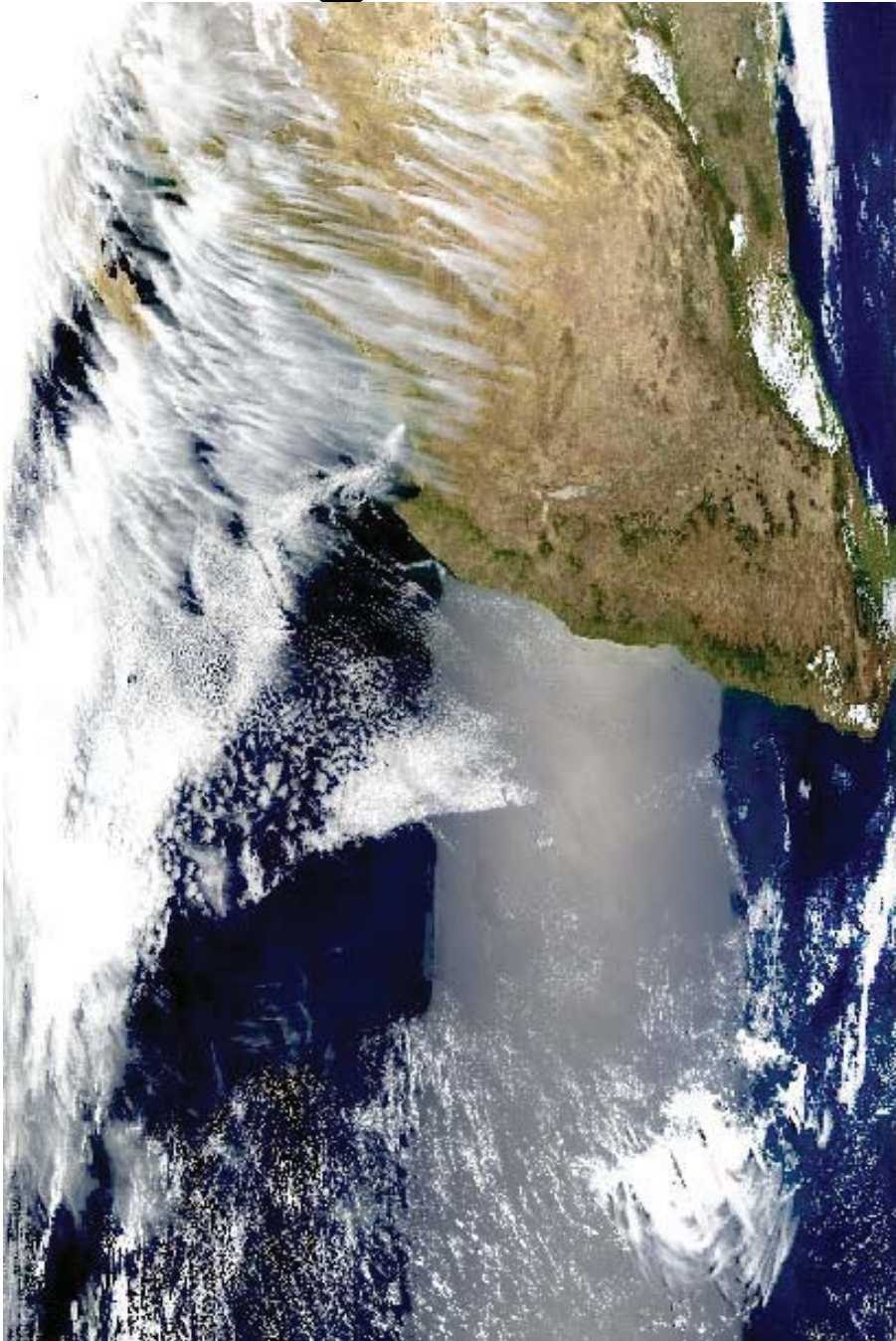
nearly diffuse reflector



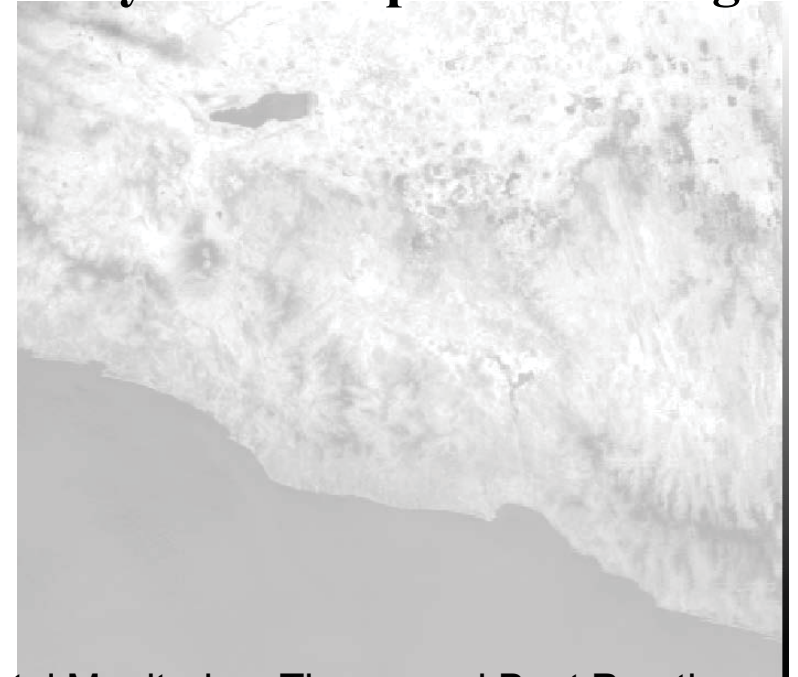
Hot spot reflection



# Sun glint as seen by MODIS



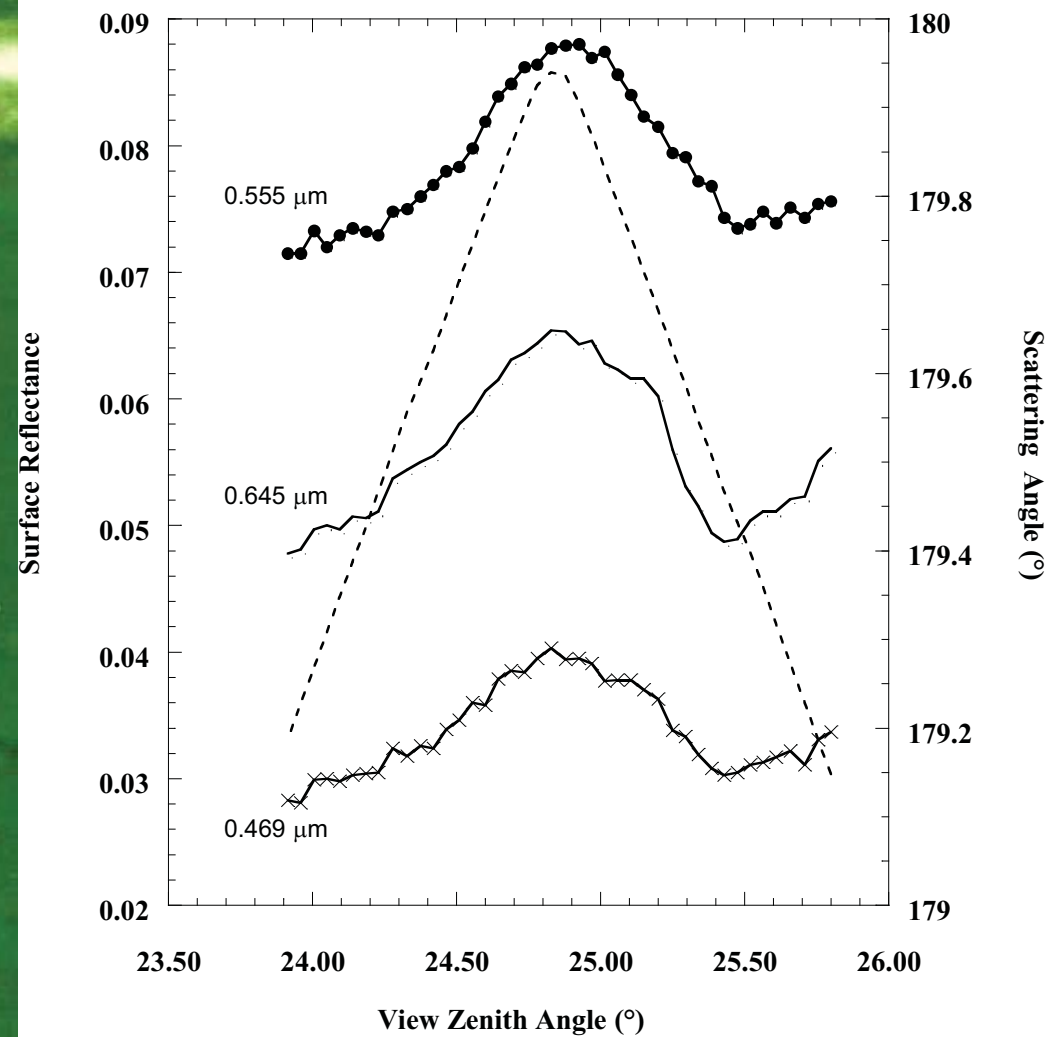
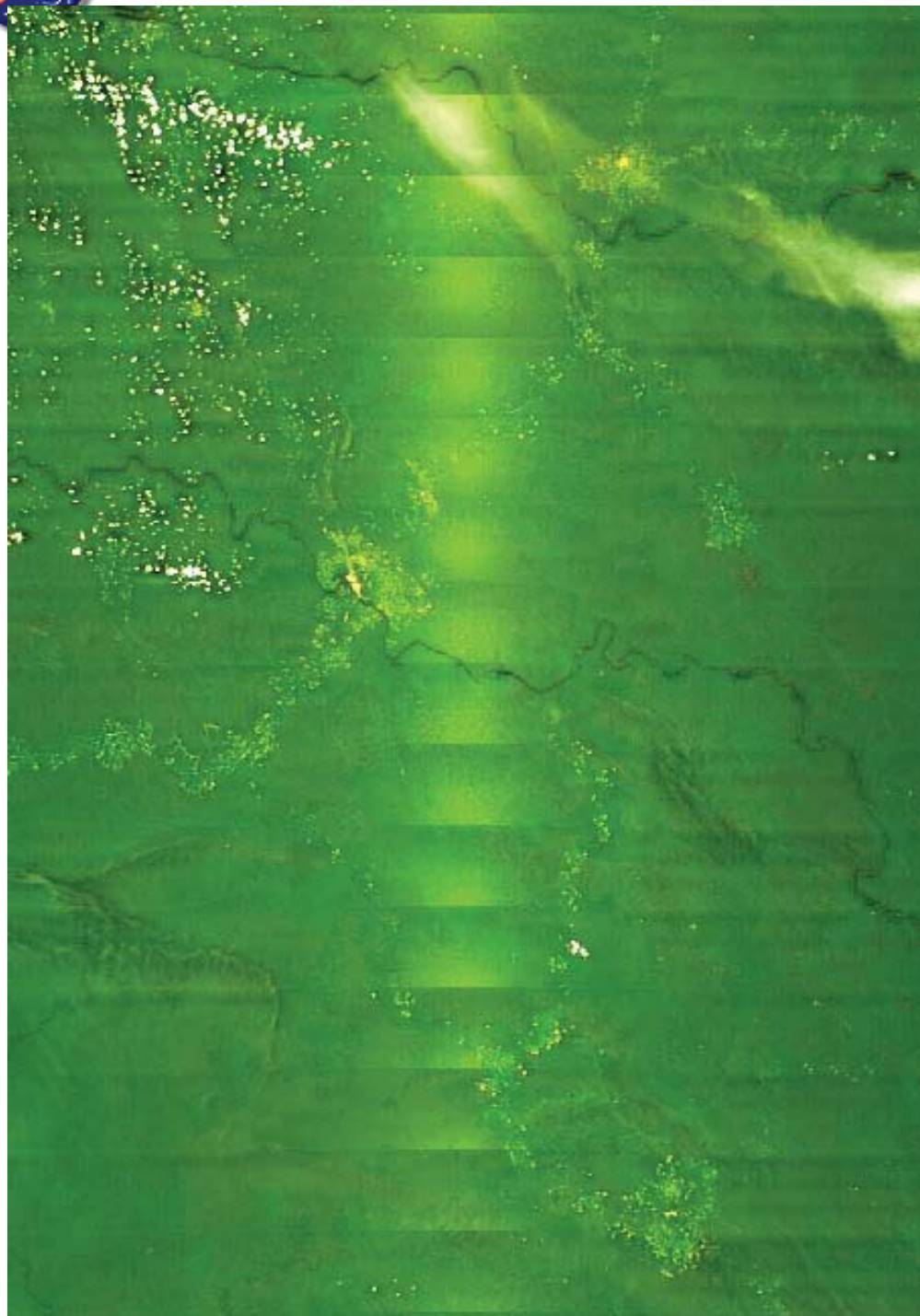
**Gray level temperature image**







## MODIS data illustrating the hot-spot over dense vegetation





# BRDF atmosphere coupling correction

## Lambertian infinite target approximation

$$\rho_{app}(\theta_s, \theta_v, \phi) = \rho_{atm}(\theta_s, \theta_v, \phi) + T_{atm}(\theta_s)T_{atm}(\theta_v) \frac{\rho_{ground}}{1 - \rho_{ground}S_{atm}}$$

## BDRF atmosphere coupling approximation

$$\begin{aligned} \rho_{app}(\theta_s, \theta_v, \phi) &= \rho_{atm}(\theta_s, \theta_v, \phi) + e^{-\tau/\mu_s} e^{-\tau/\mu_v} \rho_s(\theta_s, \theta_v, \phi) \\ &+ e^{-\tau/\mu_v} t_d(\theta_s) \bar{\rho}_s + e^{-\tau/\mu_s} t_d(\theta_v) \bar{\rho}'_s + t_d(\theta_v) t_d(\theta_s) \bar{\bar{\rho}}_s \\ &+ \frac{T_{atm}(\theta_s) T_{R+A}(\theta_v) S_{atm} (\bar{\bar{\rho}}_s)^2}{1 - S_{atm} \bar{\bar{\rho}}_s} \end{aligned}$$

$$\bar{\rho}_s(\mu_s, \mu_v, \phi) = \frac{\int_0^{2\pi} \int_0^1 \mu L_{atm}^\downarrow(\mu_s, \mu, \phi') \rho_s(\mu, \mu_v, \phi' - \phi) d\mu d\phi'}{\int_0^{2\pi} \int_0^1 \mu L_{atm}^\downarrow(\mu_s, \mu, \phi') d\mu d\phi'}$$

$$\bar{\bar{\rho}}_s(\mu_s, \mu_v, \phi) = \overline{\bar{\rho}'_s(\mu_s, \mu_v, \phi)}$$

$$\bar{\rho}'_s(\mu_s, \mu_v, \phi) = \bar{\rho}_s(\mu_s, \mu_v, \phi)$$



# Adjacency effect correction

## Lambertian infinite target approximation

$$\rho_{app}(\theta_s, \theta_v, \phi) = \rho_{atm}(\theta_s, \theta_v, \phi) + T_{atm}(\theta_s)T_{atm}(\theta_v) \frac{\rho_{ground}}{1 - \rho_{ground}S_{atm}}$$

## adjacency effect approximation

$$\rho_{app} = \rho_{atm} + \frac{T_{atm}(\theta_s)}{1 - S_{atm}\rho_e} \left( e^{-\tau/\mu_v} \rho_s + t_{atm}^d(\theta_v) \rho_e \right)$$

$$\rho_e = \frac{1}{2\pi} \int_0^{2\pi} \int_0^\infty \rho(r, \psi) \frac{dF(r)}{dr} dr d\psi$$



# Adjacency effect correction (practical implementation)

$$\rho_i = \frac{\rho_{app} - \rho_{atm}}{T_{atm}(\theta_s)T_{atm}(\theta_v)}$$

$$\rho_s^{\text{inf}} = \frac{\rho_i}{1 + S_{atm}\rho_i}$$

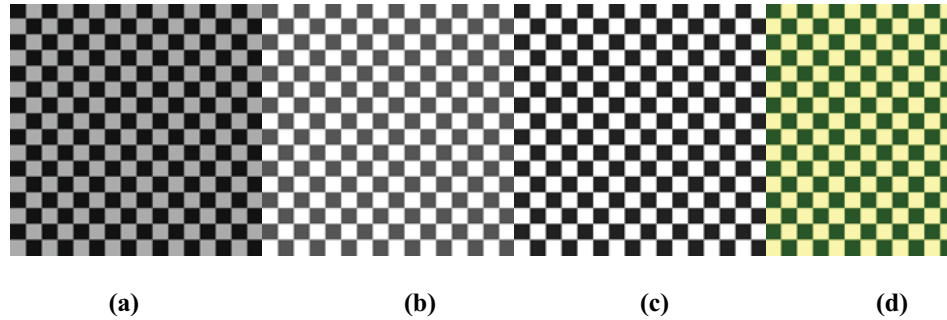
$$\rho_i = \frac{\rho_s e^{-\tau/\mu_v} + t_{atm}^d(\theta_v)\rho_e}{T_{atm}(\theta_v)(1 - S_{atm}\rho_e)}$$

$$\rho_s = \frac{\rho_i T_{atm}(\theta_v)(1 - S_{atm}\rho_e) - t_{atm}^d(\theta_v)\rho_e}{e^{-\tau/\mu_v}}$$

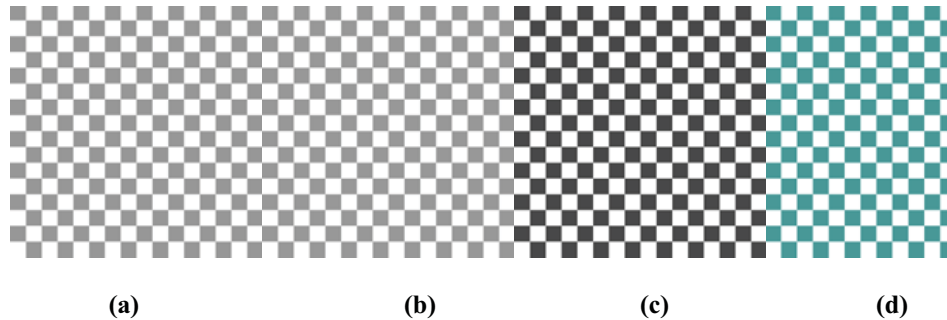
$$\rho_e = \sum_{j=-n}^n \sum_{i=-n}^n \frac{dF(r(i, j))}{dr} \rho_s^{\text{inf}}(i, j)$$



# Adjacency effect correction (testing)



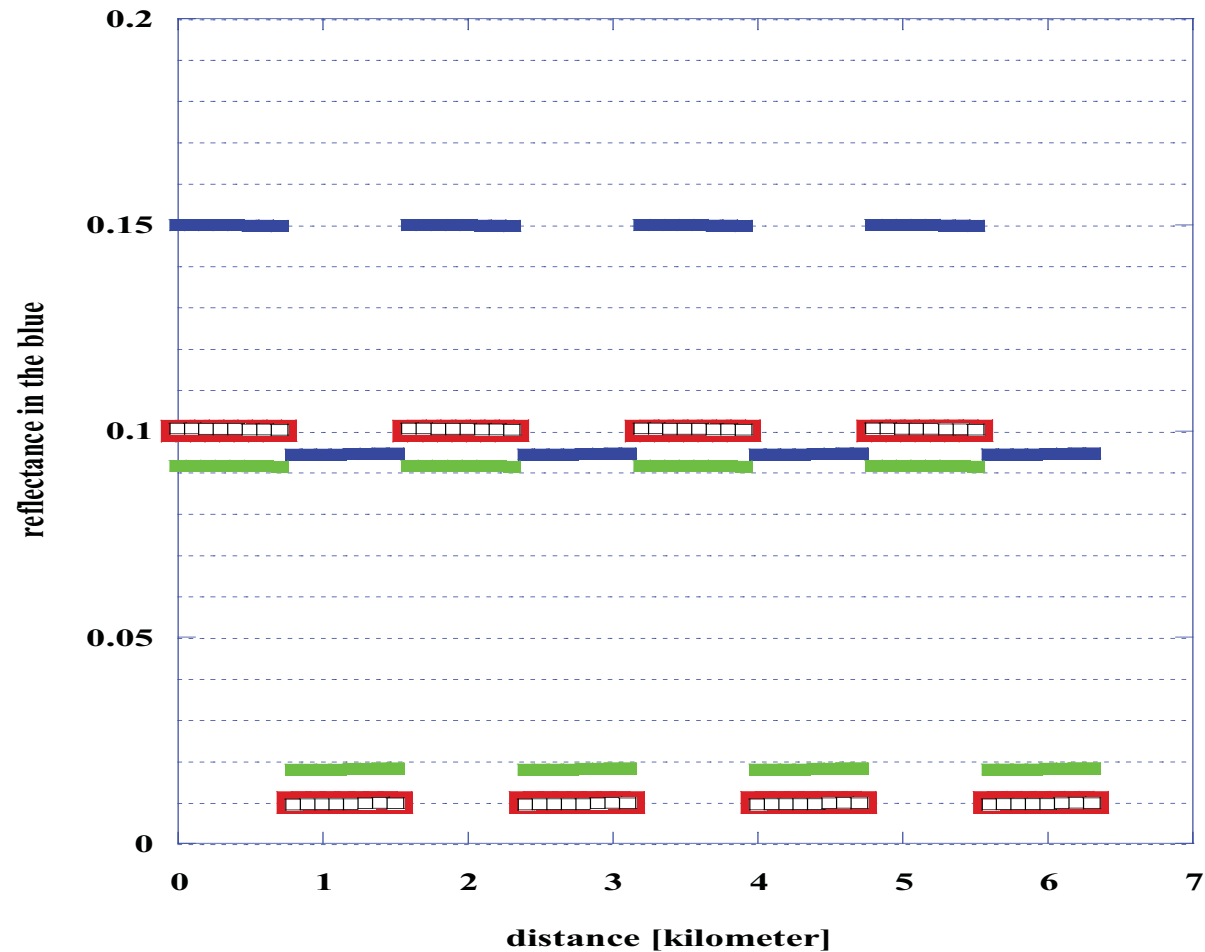
Synthetic data set for surface reflectance in the blue (a), green (b) and red (c), and in RGB (d) corresponding to bright soil (yellow squares) and dense vegetation (green squares).



Typical atmospheric effect on the synthetic surface reflectance shown above



# Adjacency effect correction (testing)



Reflectance's observed over a horizontal transect on the checkerboard. The red bars are the “true” surface reflectance, the blue bars correspond to the top of the atmosphere signal including adjacency effect. The green bars correspond to the corrected data using the infinite target assumption. The open square correspond to the data corrected for the adjacency effect using the operational method developed.





# Adjacency effect correction (validation)

